Drawing Momentum Vectors

Definition   Momentum is the product of the mass of an object and its velocity.
Equation $\vec{p} = m\vec{v}$
Units   kg•m/s
Direction   The momentum vector of an object has the same direction as the velocity of the object.
Drawing vectors When drawing a momentum vector, include reference directions and the scale on the diagram. Remember to draw the vector to scale.

1. A student on a skateboard has a combined mass of 70 kg. Calculate the momentum of the student-skateboard system if he is travelling west at 1.2 m/s. Draw the momentum vector.
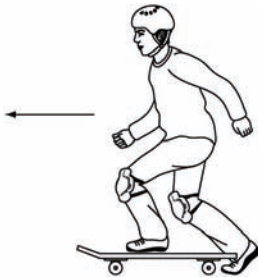
2. An athlete on a sled in the Winter Olympics skeleton event has a velocity of 35 m/s [N]. If the momentum of the athlete-sled system has a magnitude of 2.73×10^3 kg•m/s, what is the combined mass of the athlete and sled? Include a drawing of the momentum vector in your answer.
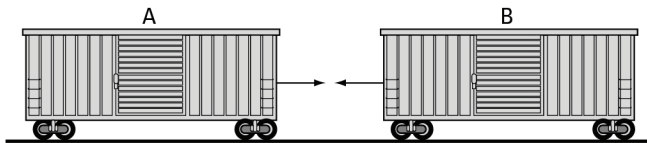
3. A 3.5-kg Canada goose has a momentum of magnitude 49 kg•m/s while flying 36º N of E.
    (a) Use a momentum vector diagram to determine the east and north components.
        Check the accuracy of your measurements by calculation.
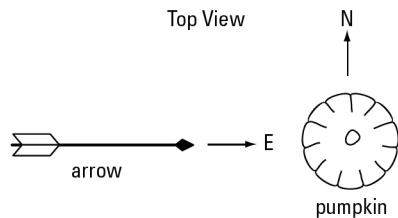   (b) What is the velocity of the goose?

Adding Momentum Vectors

Adding Vectors   Vectors may be added in any order. When adding vectors graphically, draw the tail of the next vector starting from the tip of the previous one. The resultant vector starts from the tail of the first vector and ends at the tip of the last vector.

1.   Railroad car A of mass 10 000 kg is travelling east at 0.15 m/s, when it collides with railroad car B of mass 8000 kg. Car B was initially travelling west at 0.25 m/s. The cars couple together upon impact.
     (a) Use a momentum vector addition diagram to determine the momentum of the coupled cars immediately after impact. Check the accuracy of your measurements by calculation.
     (b) Determine the velocity of the coupled cars immediately after impact.

2.   In an archery competition, a pumpkin that is suspended from a long cord is swinging north at 0.40 m/s when it is struck by a 0.050-kg arrow. The arrow becomes embedded in the pumpkin upon impact. The mass of the pumpkin is 3.0 kg, and the arrow was initially travelling east at 32 m/s.
     (a) Use a momentum vector addition diagram to determine the momentum of the arrow-pumpkin system immediately after impact. Check the accuracy of your measurements by calculation.
     (b) Determine the velocity of the arrow-pumpkin system immediately after impact.

Three-body Collisions

1. Two identical curling stones (B and C) each of mass 19.6 kg are stationary and touching each other on the ice. Another curling stone (A) moving at 1.80 m/s [S] strikes stone B off centre, and stone A moves off at 1.00 m/s [46.0º W of S] while stone B travels at 1.10 m/s [78.0º E of S]. What will be the velocity of stone C immediately after the collision? Ignore frictional and rotational effects.

2. Inside a helium balloon, helium atom A travelling at 120 m/s [35.0º] and helium atom B travelling at 100 m/s [56.0º] simultaneously collide with stationary helium atom C. Immediately after collision, atom A moves off at 110 m/s [85.8º] and atom B at 95 m/s [18.0º]. What will be the velocity of atom C immediately after the collision? The mass of each helium atom is 6.65×10^{-27} kg.

3. In front of a hockey net, a 75.0-kg centre moving at 2.50 m/s [60.0º S of E] collides with an 85.0-kg defenceman moving at 1.80 m/s [20.0º S of W]. They entangle and slide into the stationary 80.0-kg goalie, pushing him back into the crease at 1.50 m/s [74.0º S of W]. What will be the velocity of the two entangled players immediately after colliding with the goalie? Ignore any friction with the ice and rotational effects.
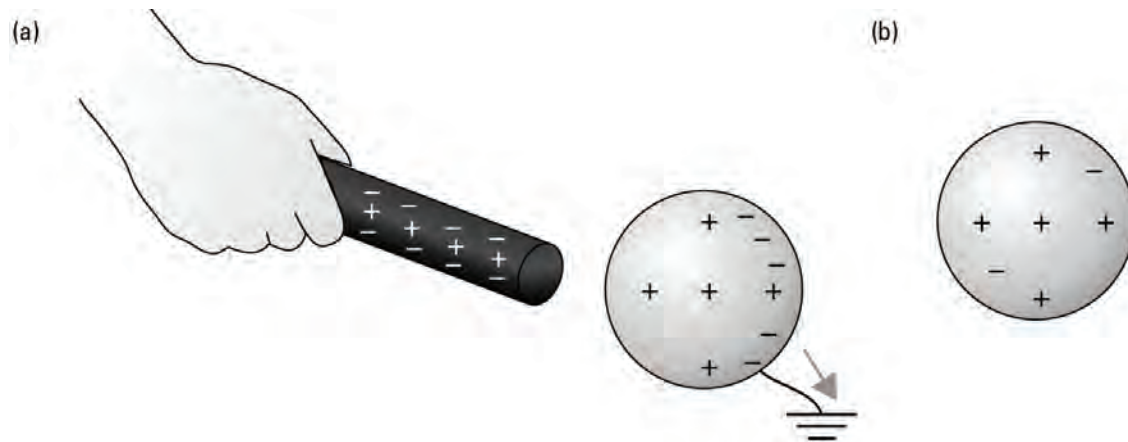
Charging by Conduction and Induction
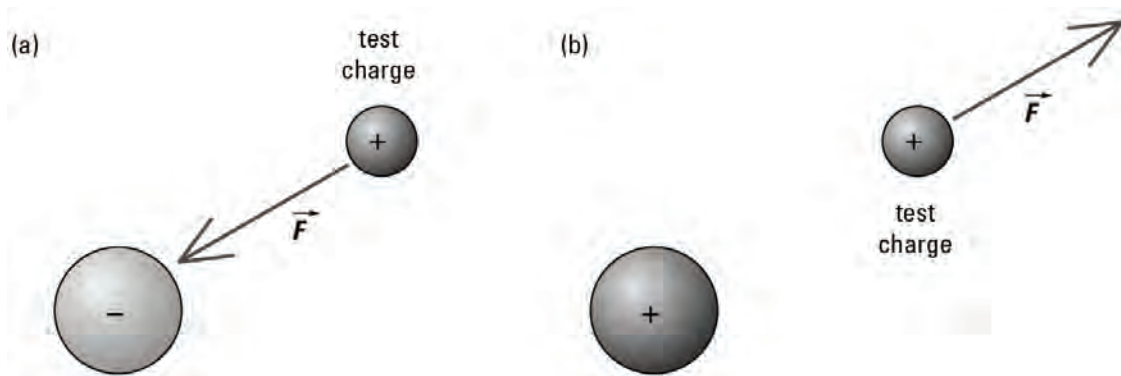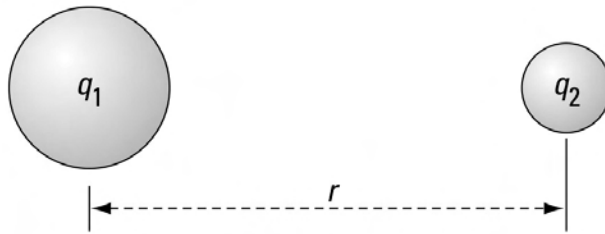
Charging by Conduction
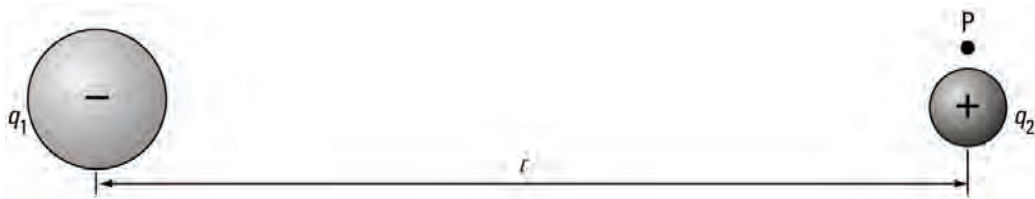
Charging by Induction
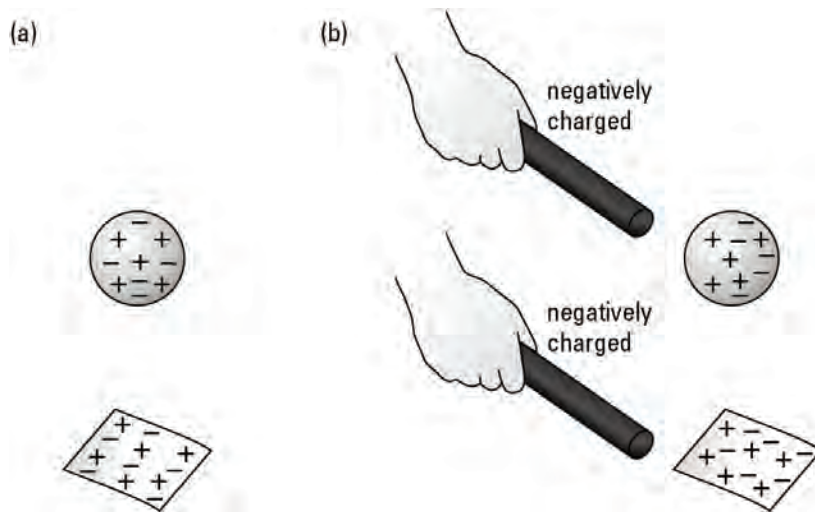
Charging by Induction with Grounding

Defining the Direction of Electric Fields

Calculating the Magnitude of the Electric Field

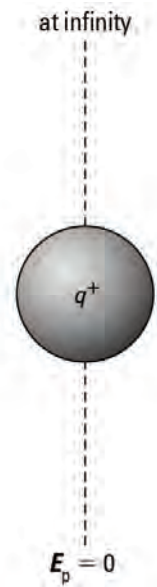$$\overset{\textbf{r}}{E} = \frac{\overset{\text{I}}{F_e}}{q}$$

$$\left| \overset{\textbf{r}}{E} \right| = \frac{kq}{r^2}$$
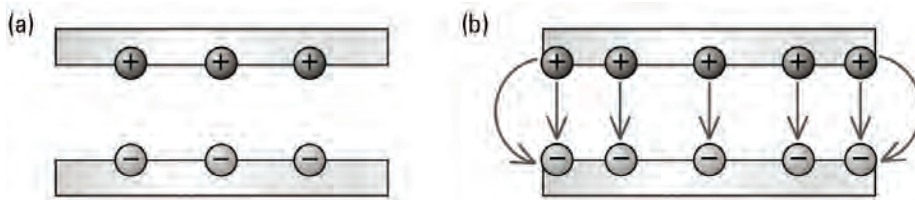
$$\left| \overset{\textbf{r}}{E} \right| = \frac{\Delta V}{\Delta d}$$

Electric Potential Energy
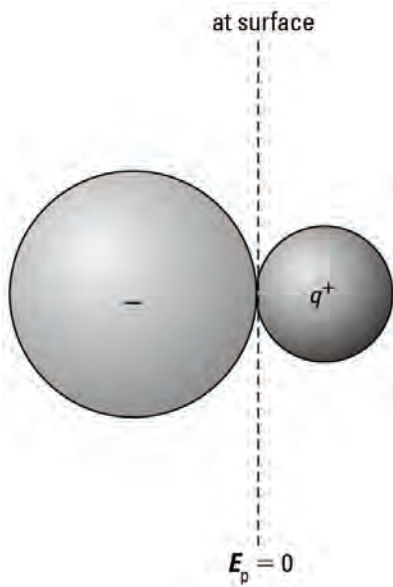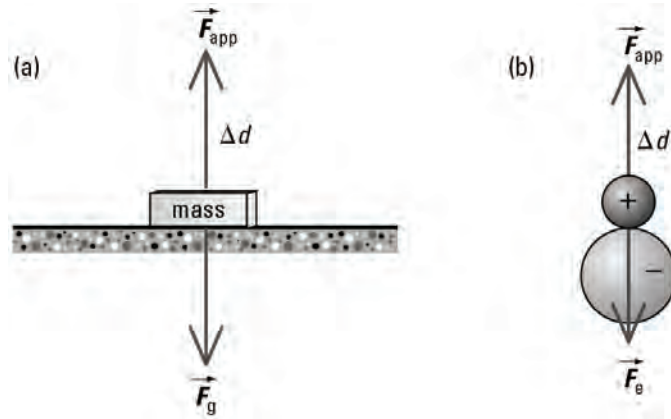
Comparing Magnetic Fields of a Bar Magnet and Earth

- A bar magnet and Earth both have magnetic fields. Draw a bar magnet like this one on the diagram of Earth, placing the poles where you think they should go so the magnetic fields match. What does your drawing show you about Earth's poles?
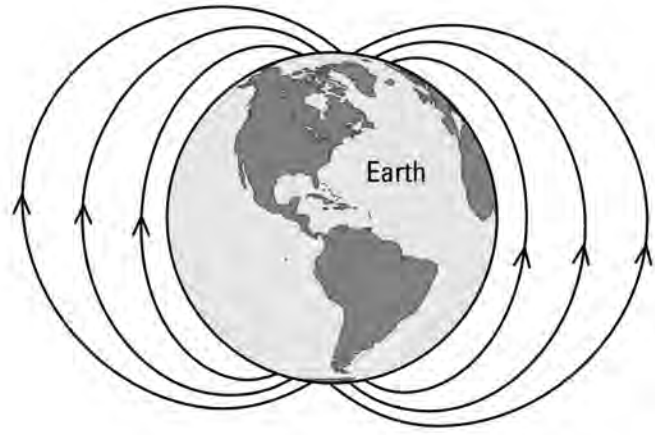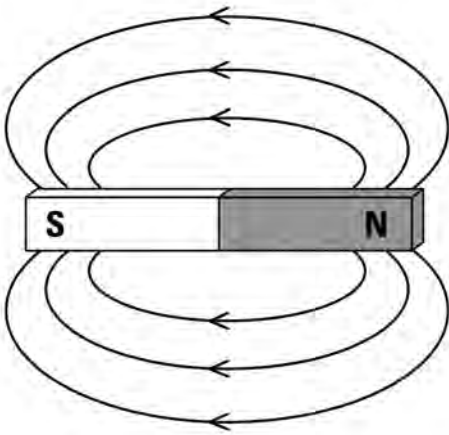
Magnetic Force on a Moving Charge and on a Conductor
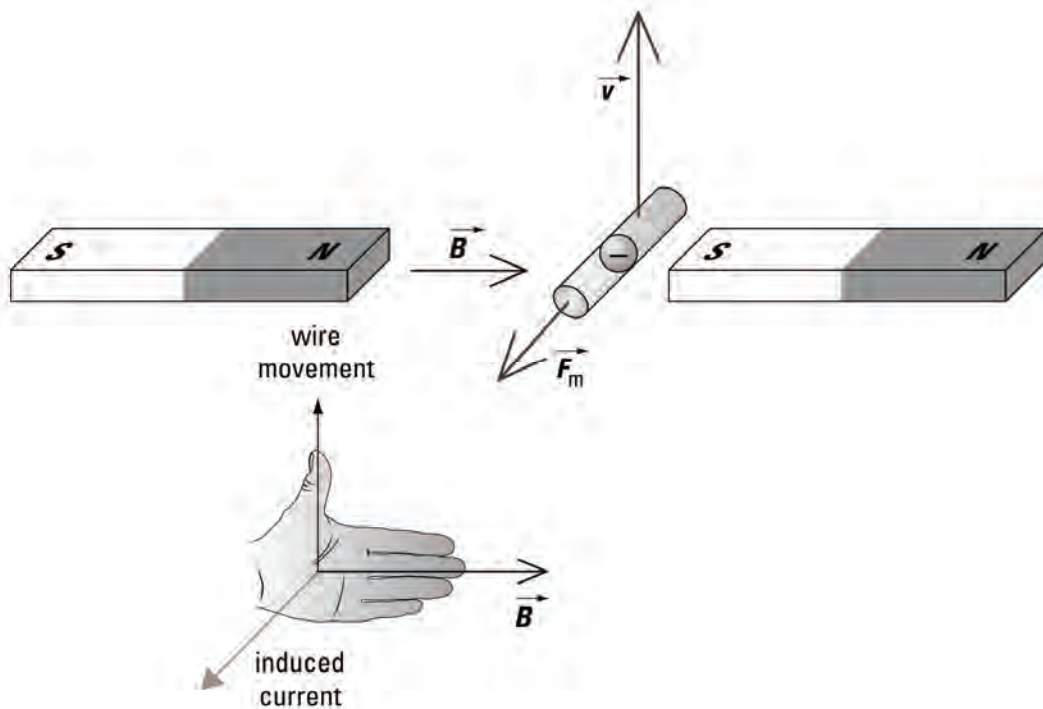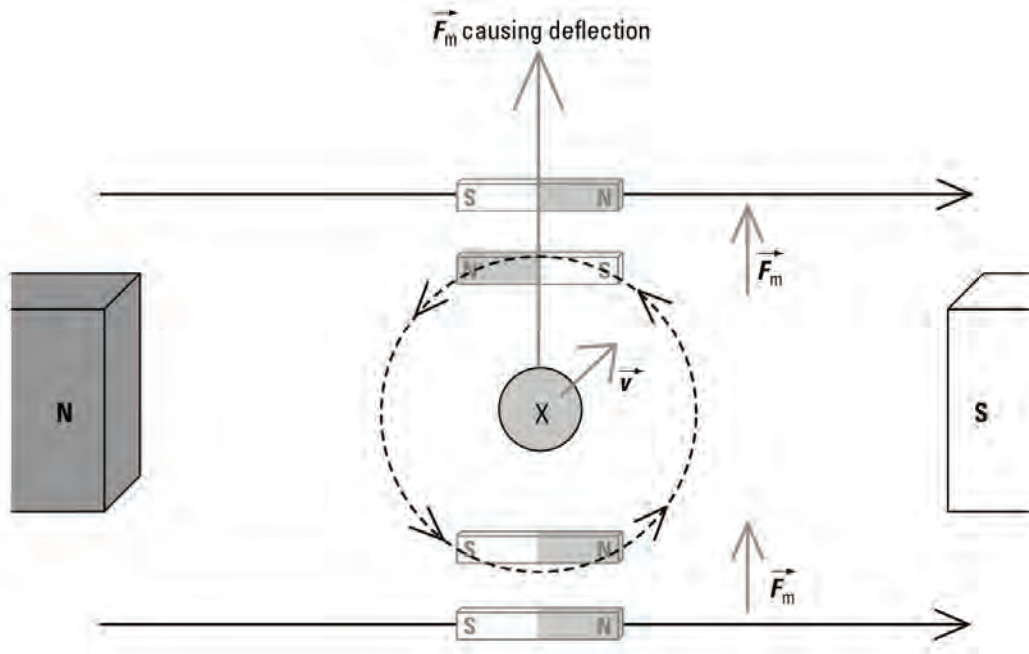
Full Spectrum
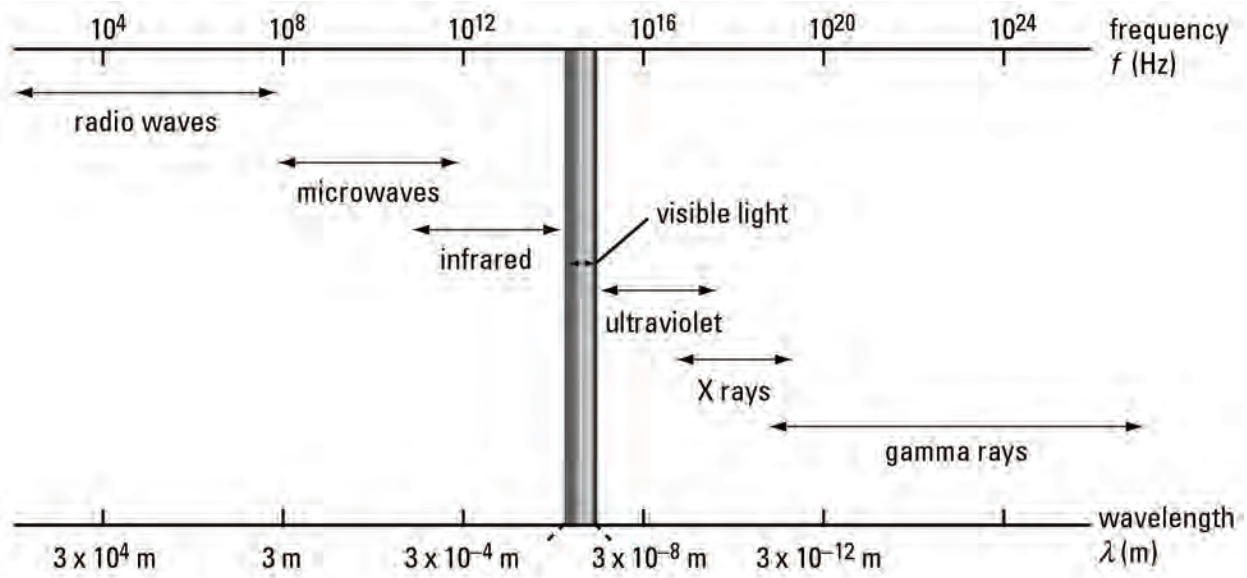
a) Electromagnetic spectrum
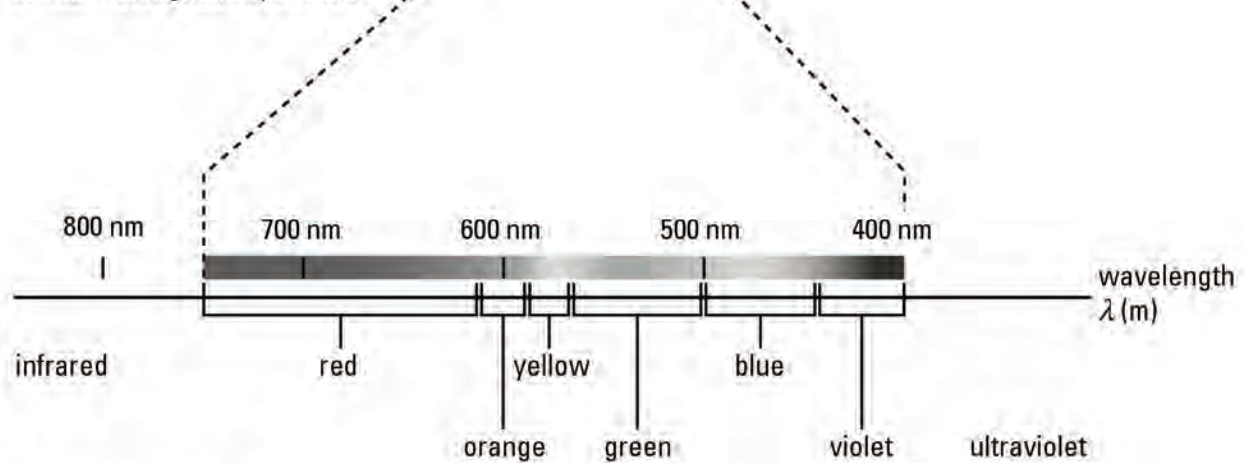
b) Visible spectrum

Hertz's Apparatus

2. Spark produces electromagnetic waves.

3. Electromagnetic waves create electric current in antenna loop; produces small spark in spark gap.

1. Induction coil produces high voltage.
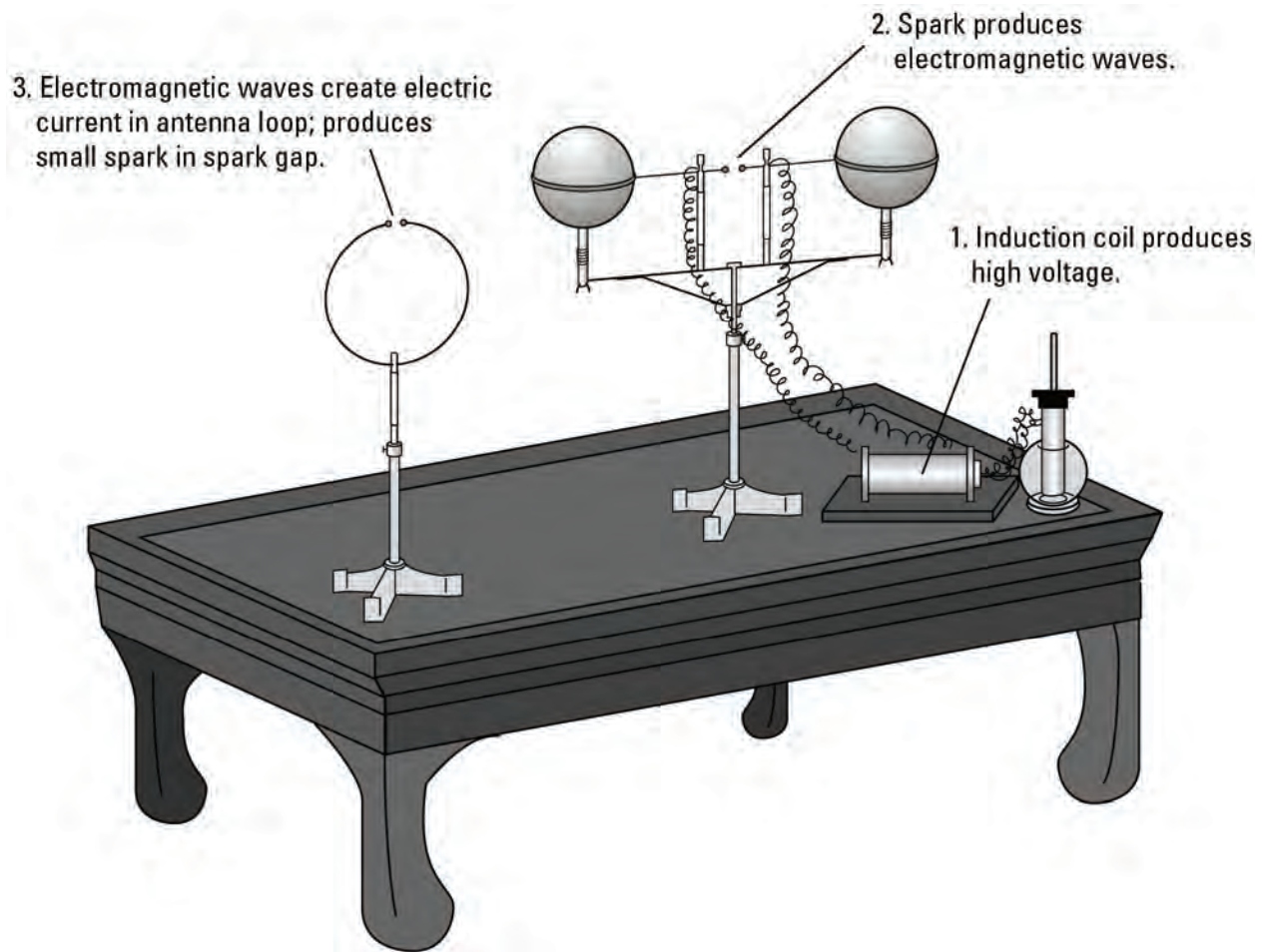
2-D Changing Electric Field

direction of
propagation
of wave

$\vec{E}$
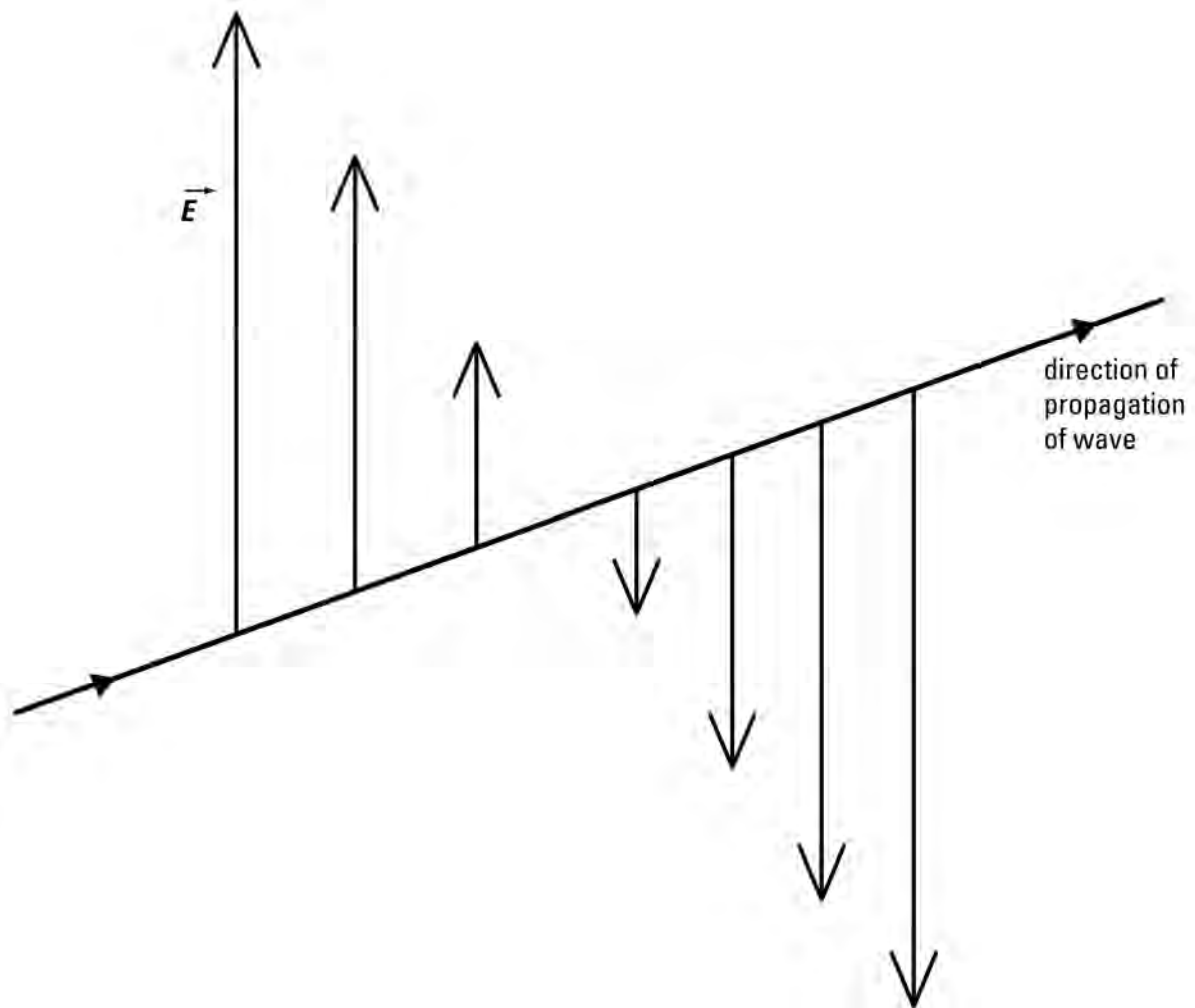
2-D Changing Magnetic Field

direction of
propagation
of wave

$\vec{B}$
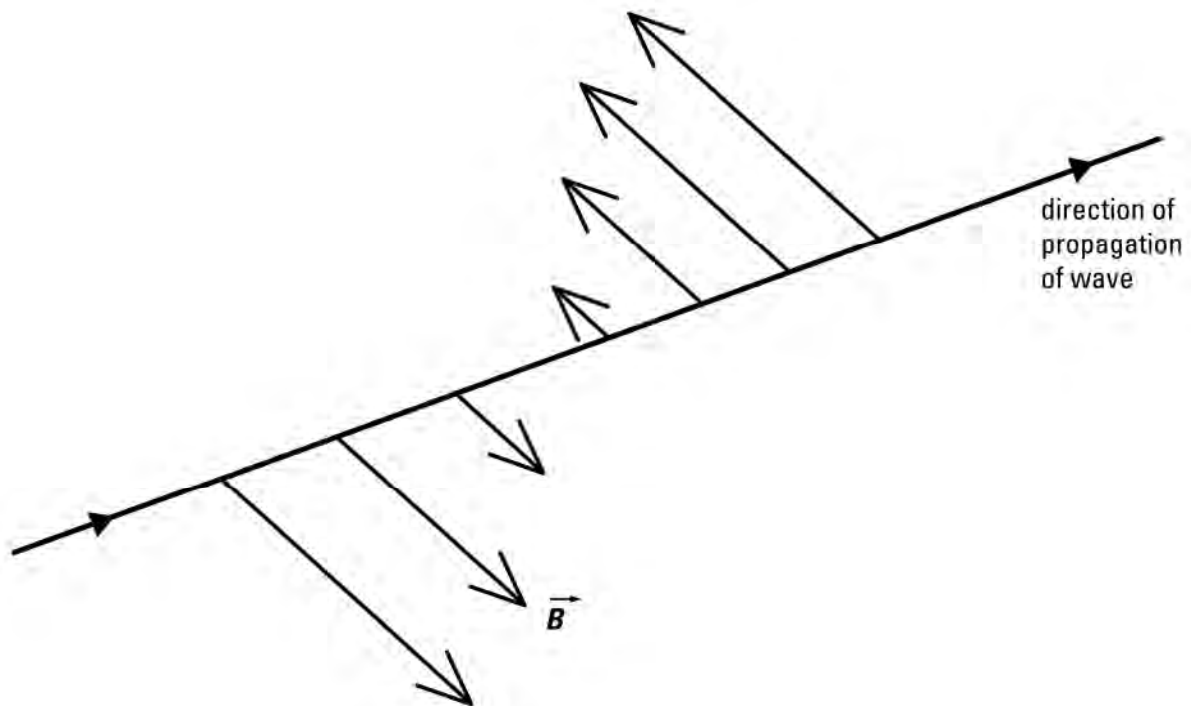
3-D EMR

Michelson Experiment

octagonal set
of mirrors

observer

telescope

Mount
Wilson

light from
source

35 km

path of light

curved mirror

Mount
St. Antonio
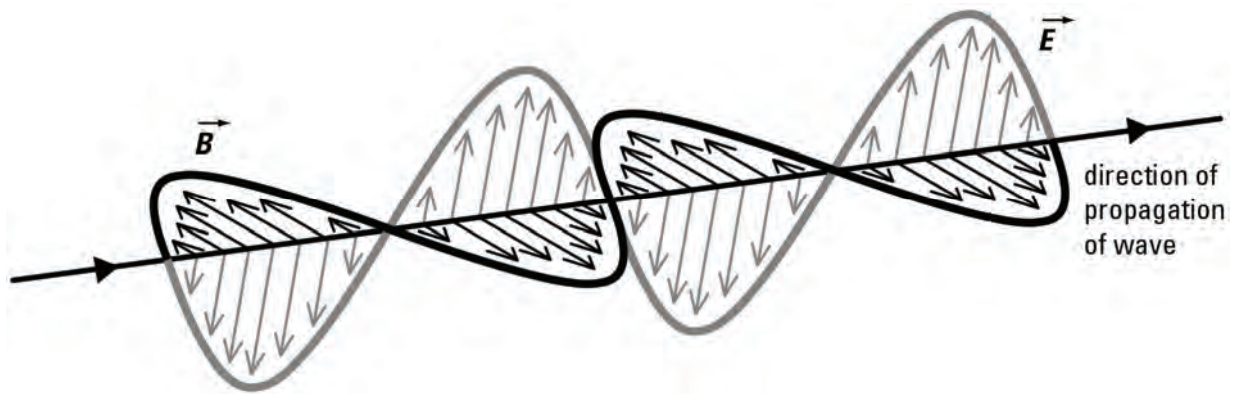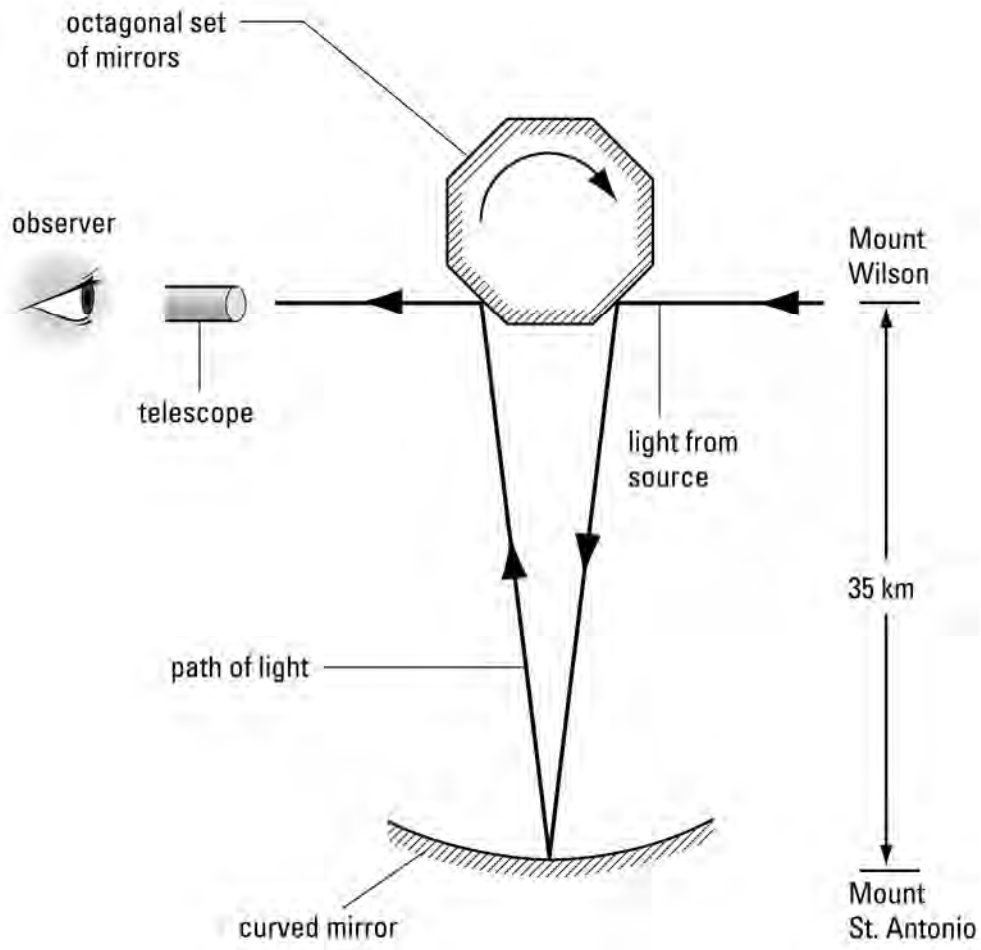
Fizeau Experiment

8

Law of Reflection

9
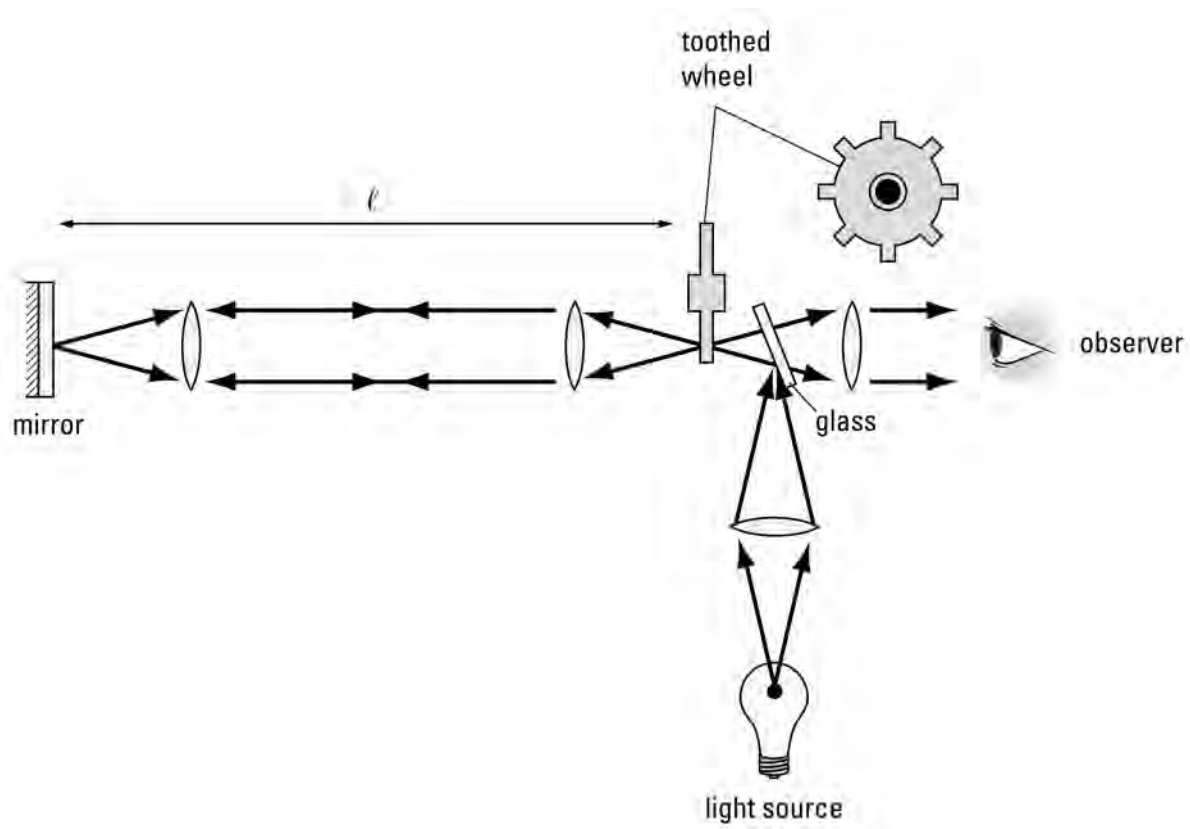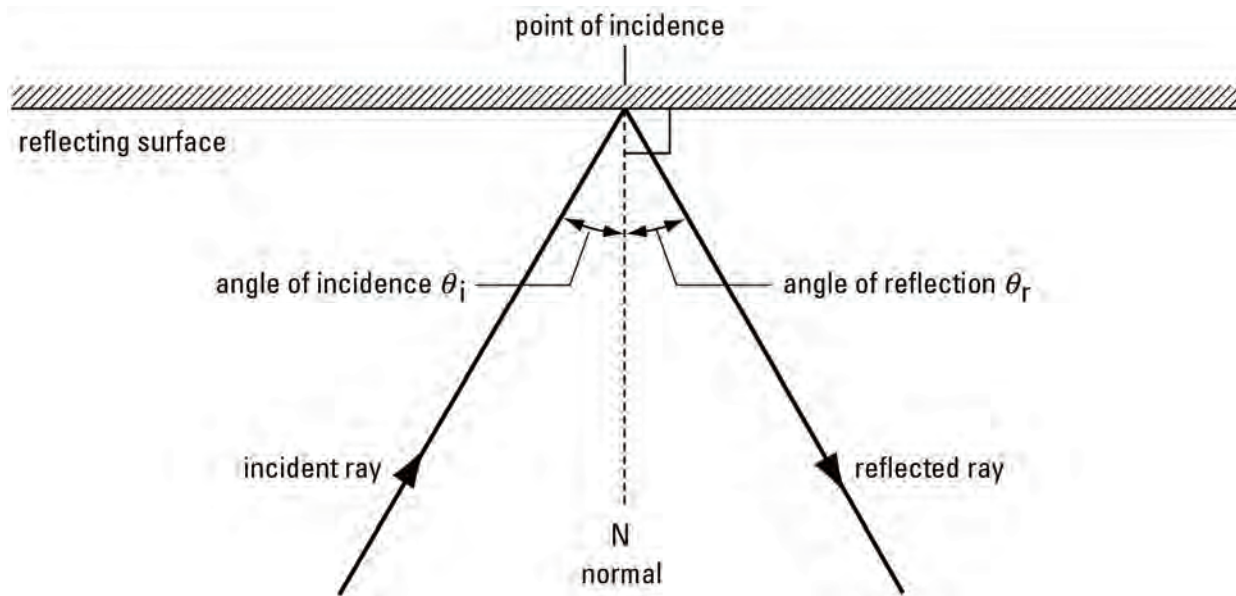
Derivation of Snell's Law

incident
wave fronts

λ_1

vacuum speed = c

B

$\Delta d = c\Delta t$

less refractive medium
$n = 1.00$

A

i

D

more refractive
medium $n > 1.00$

$\Delta d = v\Delta t$

R

refracted
wave fronts

C

λ_2

medium speed = v

Enlarged View

incident ray

incident wave front

less optically
dense medium

θ_1

λ_1

θ_1 x

more optically
dense medium

λ_2

θ_2

refracted
wave front

θ_2

refracted
ray
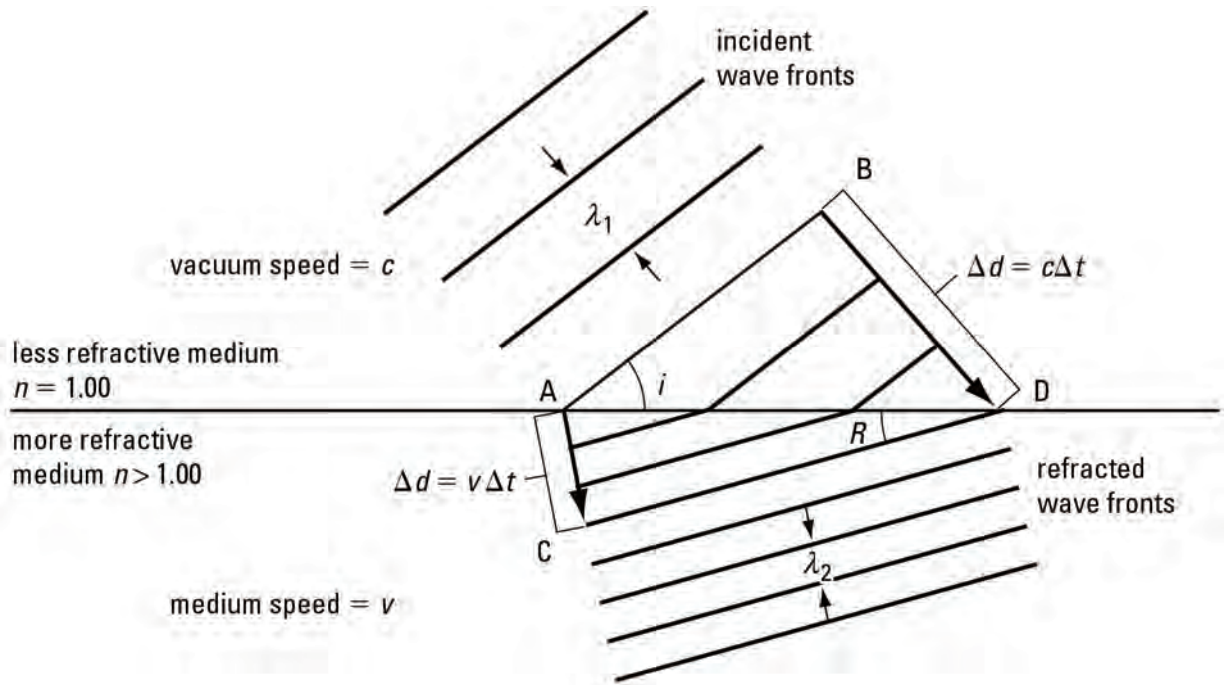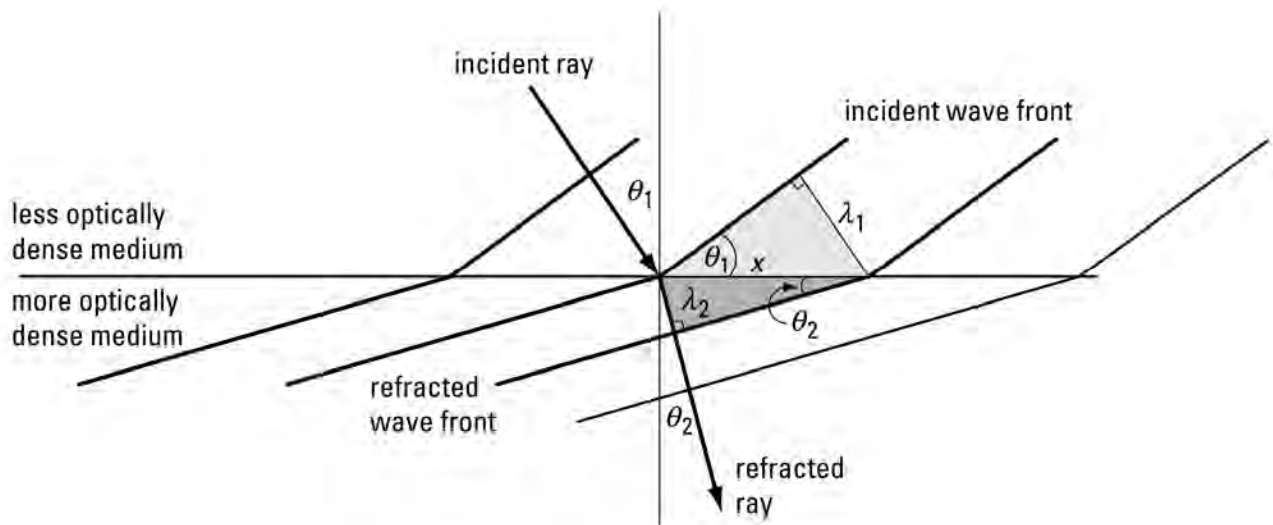
Path Differences Causing Interference

path difference:
$3.00 \lambda - 3.00 \lambda = 0 \lambda$

path length = 3.00λ

screen

d

constructive interference at central antinode

path length = 3.00λ

l

path difference:
$3.75 \lambda - 3.25 \lambda = \frac{1}{2} \lambda$

path length = 3.75λ

d

destructive interference at 1st node

path length = 3.25λ

l

path difference:
$4.50 \lambda - 3.50 \lambda = 1 \lambda$

path length = 4.50λ

d

constructive interference at $n = 1$ antinode

path length = 3.50λ

l
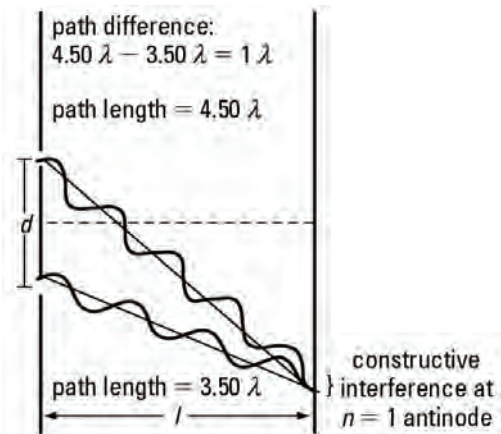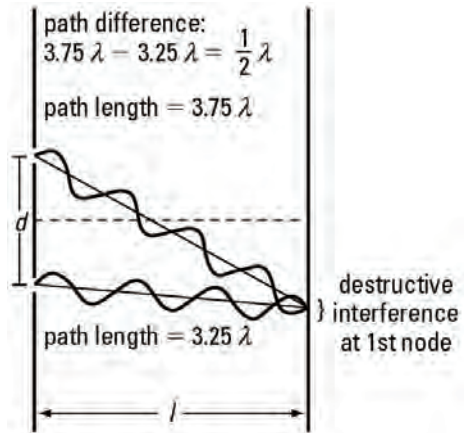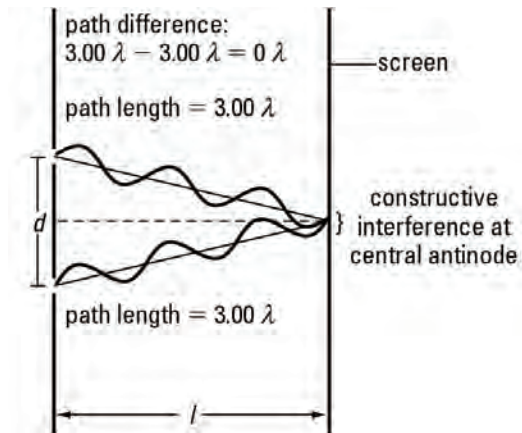
Two Point Sources and Interference Patterns

dark nodes
(destructive
interference)

bright antinodes
(constructive
interference)

bright central
antinode

4   3   2   1   1   2   3   4   screen

bisector

10λ 9λ

P

S_1 S_2

d
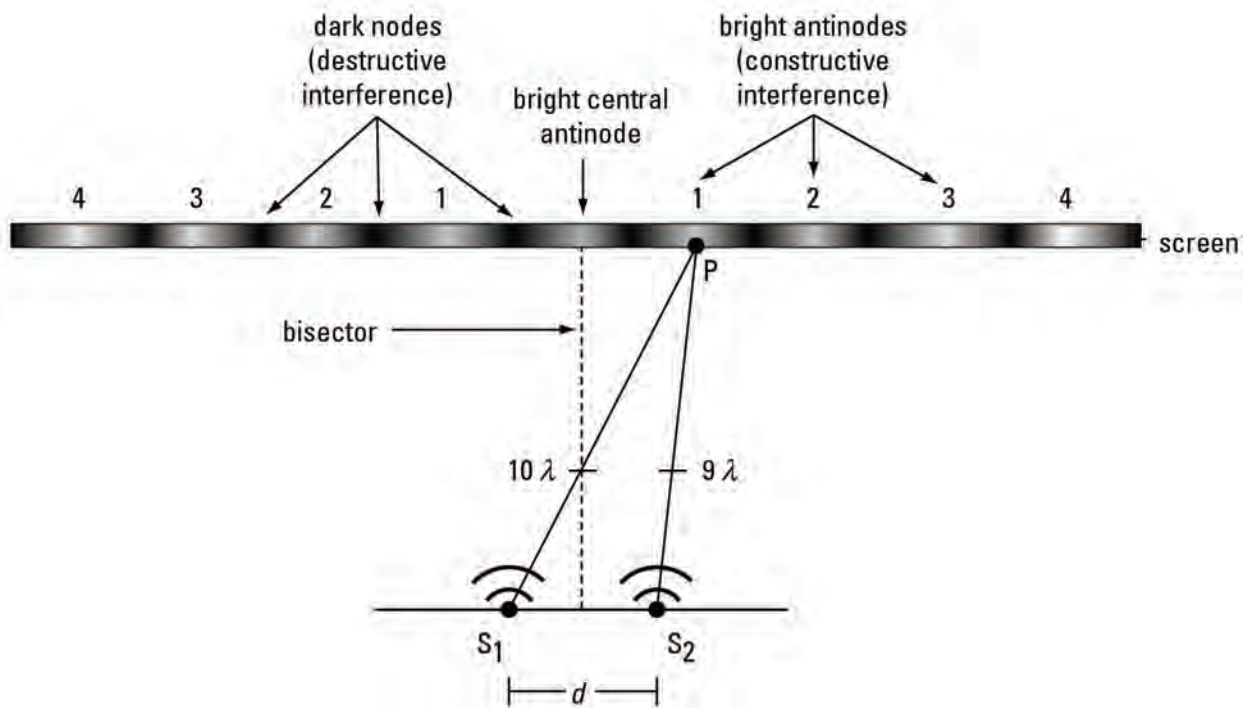
The Puzzling Results of Early Research on the Photoelectric Effect

- The number of electrons released from a metal surface increases as the intensity of the light is increased, but the energies of the emitted electrons do not depend on the intensity of the light.

- No electrons are emitted from the surface of a metal unless the frequency of the light is greater than a certain minimum value. When electrons are ejected from the surface, they exhibit a range of kinetic energies, from zero up to some maximum value.

- The energy of the electrons with the maximum velocity is found to increase linearly with an increase in the frequency of the incident light.

- When electrons are emitted, they are emitted almost instantaneously.

Millikan's Thoughts on the Results of his Work on the Photoelectric Effect

"Einstein's explanation of 1905 ignored and indeed seemed to contradict all the manifold facts of interference and thus seemed to be a straight return to the corpuscular theory of light which had been completely abandoned since the time of Young and Fresnel….
I spent 10 years of my life testing the 1905 equation of Einstein's and, contrary to all my expectations, I was compelled in 1915 to assert its unambiguous experimental verification in spite of all its unreasonableness since it seemed to violate everything we knew about the interference of light."

Things to Think About …

- Are scientists dispassionate observers of the world, or do they come to their experiments with their own beliefs and prejudices? What does Millikan's quote suggest about this question?
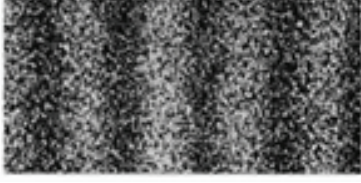
- Would a classically trained physicist have found the particle model of light easy or difficult to accept in 1915? Why or why not?

The Results of Millikan's Experiment

Maximum E_k vs. Frequency
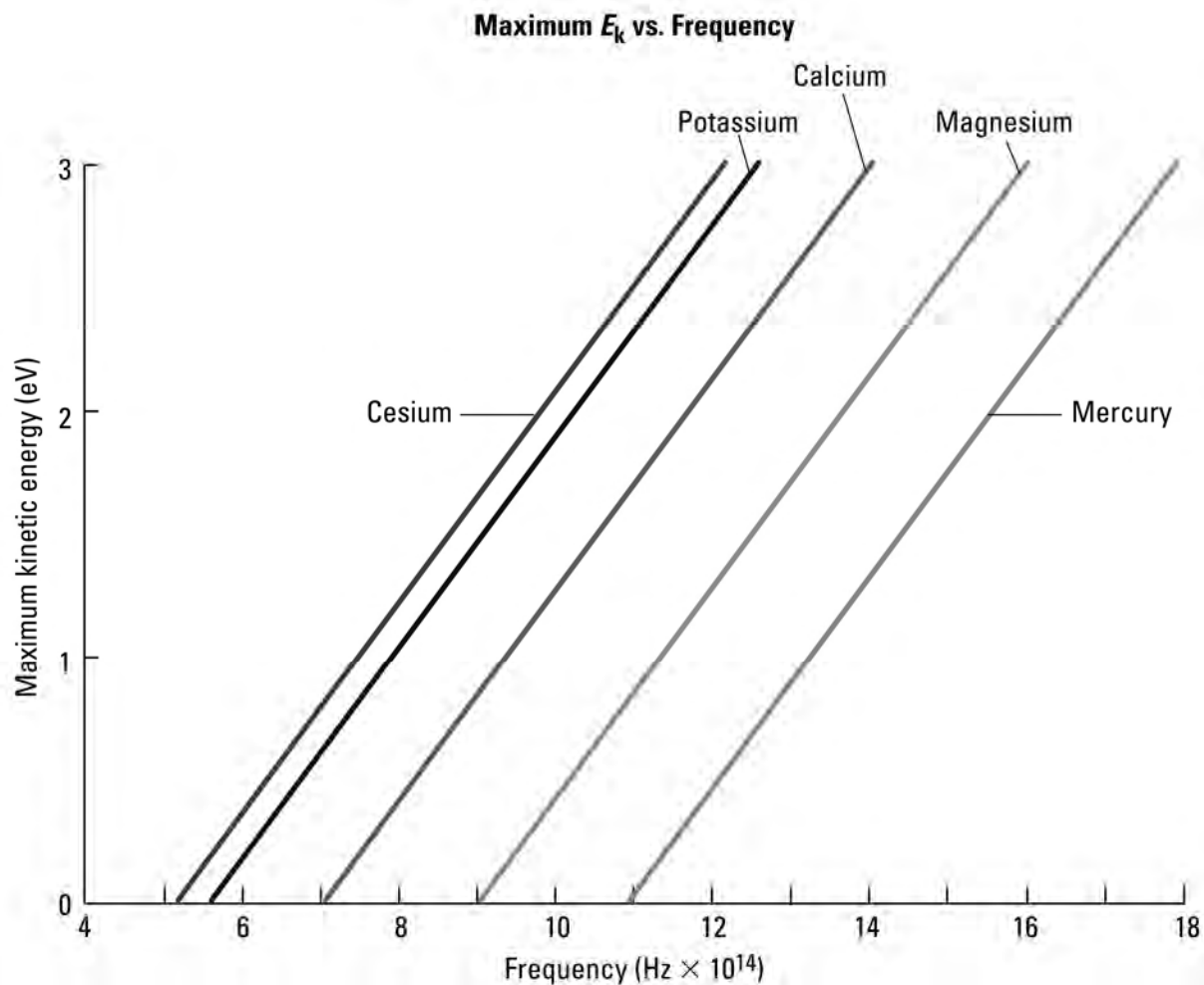
1. Why do all these graphs have the same slope?

2. Why do these graphs have different *x*-intercepts? What does this mean?

Photon (or Electron) Interference

	Only a few photons (or electrons) have passed through the slits.
	Several hundred photons (or electrons) have passed through the slits, but there is as yet no obvious pattern.
	Can you see the formation of nodes?
	The pattern is becoming more evident.
	The pattern is identical to the double-slit pattern for light seen by Thomas Young in 1800!

Richard Phillips Feynman (1918–1988)

"It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong."

"The first principle is that you must not fool yourself and you are the easiest person to fool."

"I think I can safely say that no one understands quantum mechanics."

"The electron does anything it likes …. It just goes in any direction at any speed, forward or backward in time, however it likes, and then you add up the amplitudes and it gives you the wave function."

"I can live with doubt and uncertainty and not knowing. I think it is much more interesting to live not knowing than to have answers that might be wrong."

"Our imagination is stretched to the utmost, not, as in fiction, to imagine things which are not really there, but just to comprehend those things which are there."

A Cathode-ray Tube with a Magnetic Field in Different Orientations
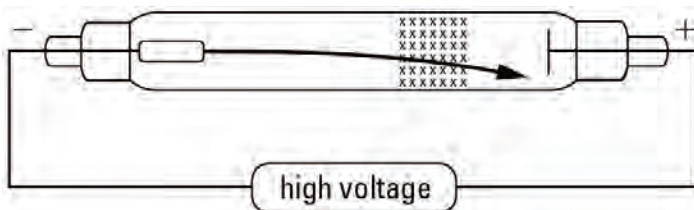
The magnetic field is oriented into the cathode-ray tube:
- Apply the hand rules.
- $F_{magnetic} = qBv\sin(90°)$
- The electrons are deflected downward.
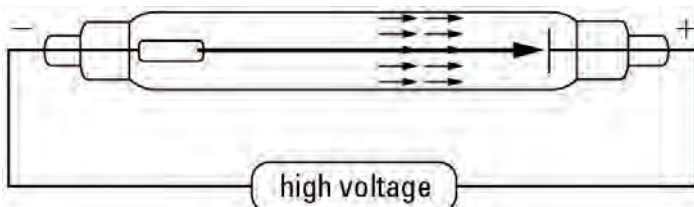
The magnetic field is oriented parallel to the cathode-ray tube:
- Apply the hand rules.
- $F_{magnetic} = qBv\sin(0°)$
- There is no deflection.
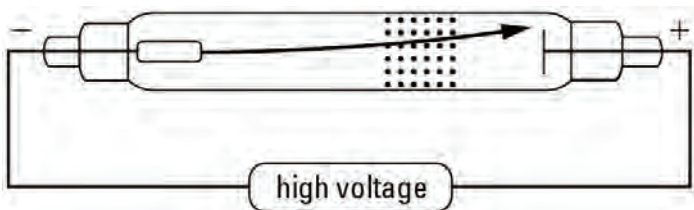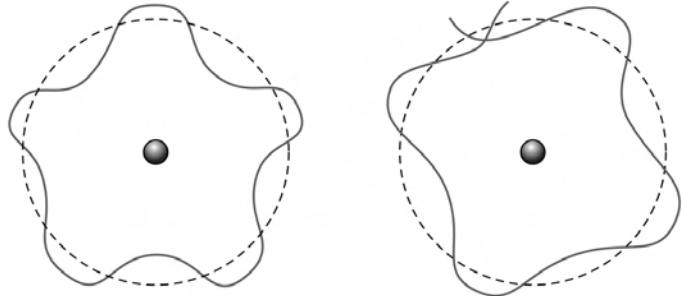
The magnetic field is oriented out of the cathode-ray tube:
- Apply the hand rules.
- $F_{magnetic} = qBv\sin(90°)$
- The electrons are deflected upward.

Contrasting Energy Level Transitions Using Scale and Energy Level Diagrams

- A scale energy level diagram for hydrogen accurately shows that the spacing between successive energy levels decreases as n increases.

- $E_n = -\dfrac{E_1}{n^2}$, where E_1 is the ground-state energy.

- Transitions that end in the $n = 1$ or ground-state energy level undergo the greatest energy change and thus involve the most energetic photons.

- This diagram is a simpler but less accurate way to illustrate energy levels. It does not illustrate the inverse square relationship with n.

Standing Waves and the Hydrogen Atom
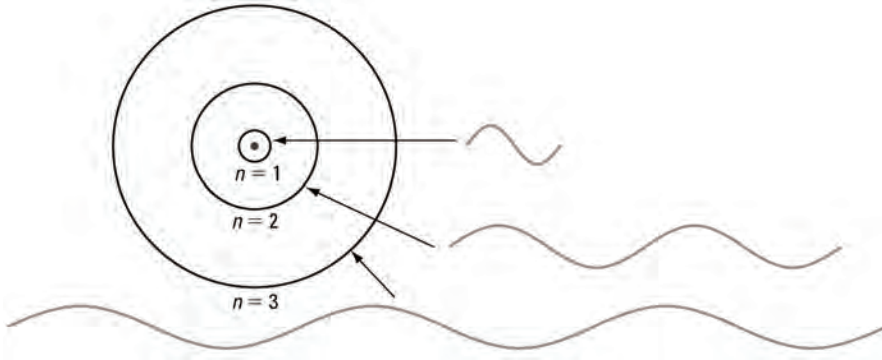
constructive
interference

destructive
interference

A standing wave is
possible only if a
whole number of
electron wavelengths
fits exactly along the
circumference of
an orbit.

$n = 1$
$n = 2$
$n = 3$

Standing waves in the
first three energy
levels of the hydrogen
atom imply that
$$2\pi r_n = n\lambda \text{ .}$$

Stability of Isotopes Graph
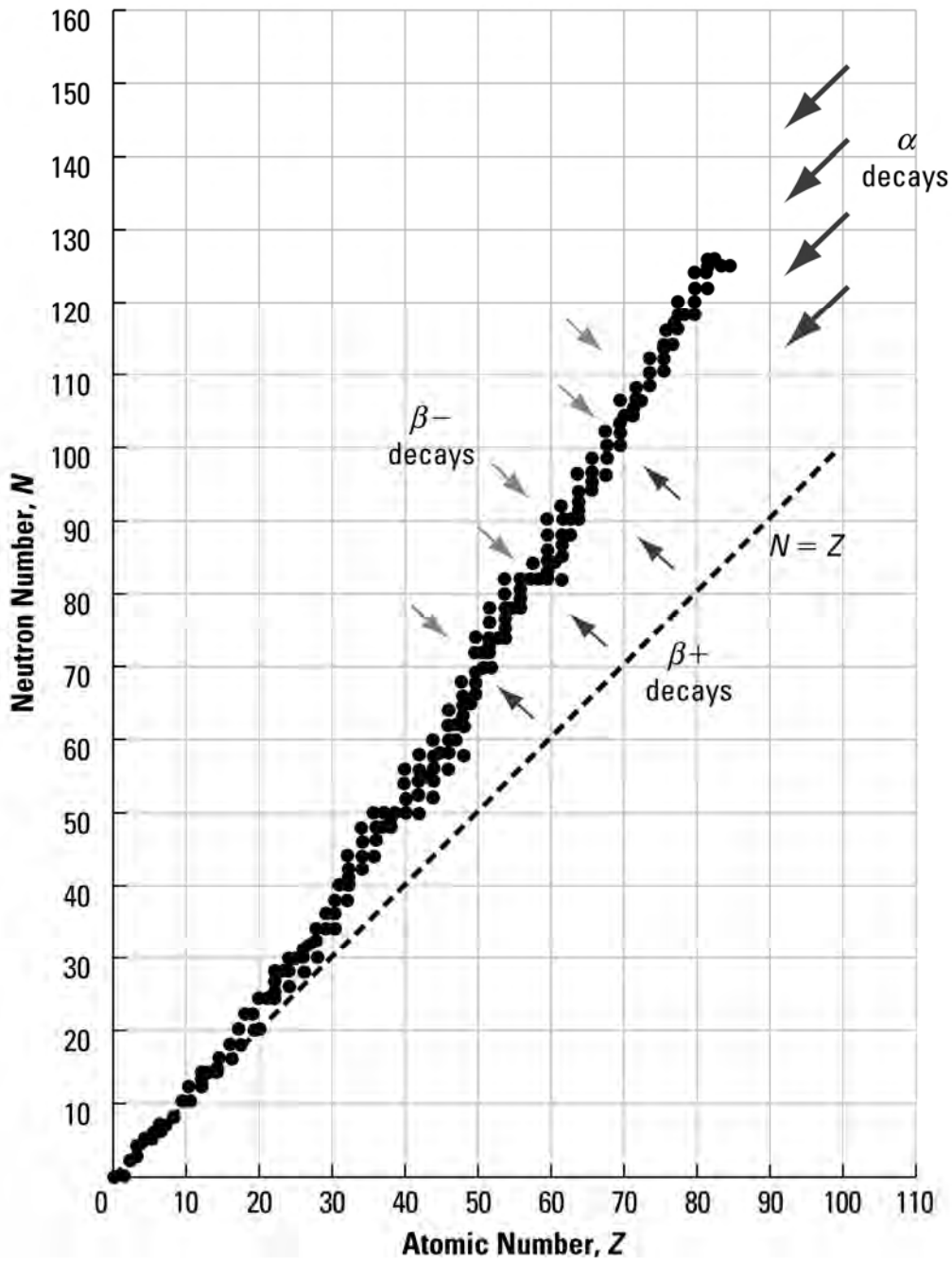
Alpha Decay
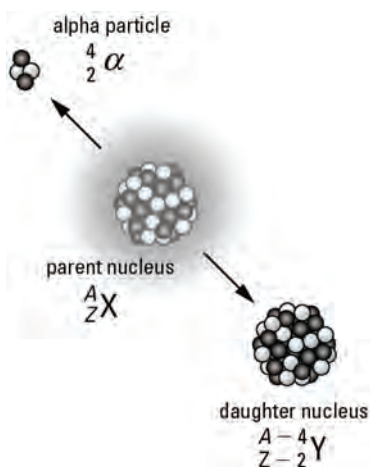
- Alpha decay is one of the three main forms of radioactive decay. Sometimes, an unstable nucleus can become more stable by emitting an alpha particle, which is the same as a helium nucleus. The process of alpha decay also forms a new nucleus, called the daughter nucleus.
- The alpha particle and the daughter nucleus conserve both momentum and energy during the decay process.
- The basic equation for alpha decay is $^{A}_{Z}X \rightarrow \,^{A-4}_{Z-2}Y + \,^{4}_{2}\alpha$.

Two Forms of Beta Decay
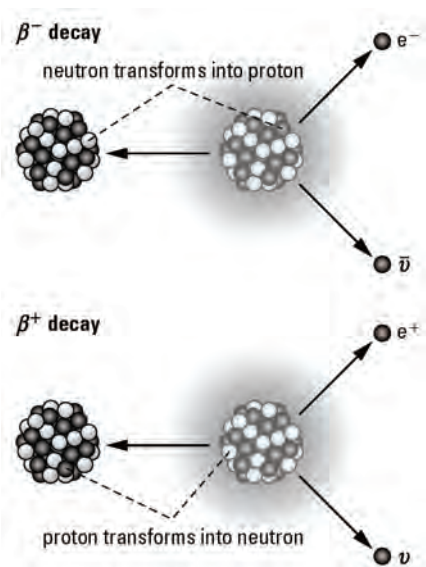
During β^- decay, a neutron in the nucleus decays into a proton and an electron. The nucleus emits an electron and a neutrino. Because a new positive charge appears in the nucleus, the atomic number increases by one unit.

$$_Z^A X \rightarrow \, _{Z+1}^{A} Y + \, _{-1}^{0} \beta + \overline{\nu}$$
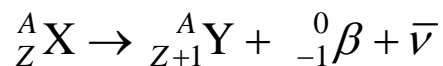
During β^+ decay, a proton in the nucleus decays into a neutron and a positron. The nucleus emits a positron and a neutrino. Because a positive charge disappears in the nucleus, the atomic number decreases by one unit.

$$_Z^A X \rightarrow \, _{Z-1}^{A} Y + \, _{1}^{0} \beta + \nu$$
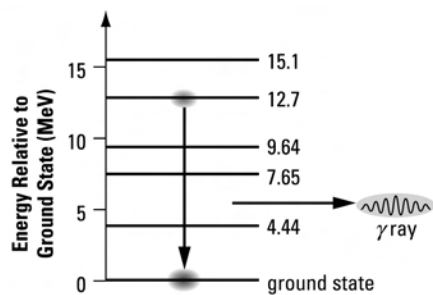
Gamma Radiation

A nucleus emits gamma radiation when a nucleon drops from an excited energy level in the nucleus to a lower energy level. In the diagram, the nucleon drops from a level of 12.7 MeV to the ground state, 0 MeV. This drop results in the emission of a 12.7-MeV gamma ray.

Gamma-ray emission does not change the mass or the atomic number of the nucleus.

$$_Z^A X^* \rightarrow \ _Z^A X + \gamma$$

The Biological Effects of Radiation Exposure

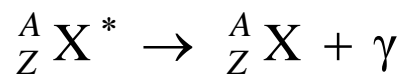| Exposure Range (Sv)	Symptoms
0.05–0.2	No symptoms; potential for cancer and mutation of genetic material
0.2–0.5	No noticeable symptoms; red blood cell count decreases temporarily
0.5–1	Mild radiation sickness with headache and increased risk of infection due to disruption of immunity cells; temporary male sterility is possible.
1–2	*Light radiation poisoning, 10% fatality after 30 days (LD 10/30).* Typical symptoms include mild to moderate nausea (50% probability at 2 Sv), with occasional vomiting, beginning 3–6 h after irradiation and lasting for up to one day. After one day, light symptoms manifest, such as general illness, anorexia, and fatigue (50% probability at 2 Sv). The immune system is depressed, so there is extended convalescence and increased risk of infection. Temporary male sterility is common.
2–3	*Severe radiation poisoning, 35% fatality after 30 days (LD 35/30).* Nausea is common (100% at 3 Sv), with 50% risk of vomiting at 2.8 Sv. Symptoms onset occurs 1–6 h after irradiation and symptoms last for 1–2 days. After that, the following symptoms appear: loss of hair all over the body (50% probability at 3 Sv), fatigue, and general illness. There is a massive loss of leukocytes, greatly increasing the risk of infection. Permanent female sterility is possible. Convalescence takes one to several months.
3–4	*Severe radiation poisoning, 50% fatality after 30 days (LD 50/30).* Other symptoms are similar to the 2–3 Sv dose, with uncontrollable bleeding in the mouth, under the skin, and in the kidneys (50% probability at 4 Sv).
4–6	*Acute radiation poisoning, 60% fatality after 30 days (LD 60/30).* Fatality increases from 60% at 4.5 Sv to 90% at 6 Sv (unless there is intense medical care). Symptoms start 0.5–2 h after irradiation and last for up to 2 days. Female sterility is common at this point. Convalescence takes several months to a year. The primary causes of death (in general, 2–12 weeks after irradiation) are infections and internal bleeding.
6–10	*Acute radiation poisoning, 100% fatality after 14 days (LD 100/14).* Survival depends on intense medical care. Bone marrow is nearly or completely destroyed, so a bone marrow transplant is required. Gastric and intestinal tissue is severely damaged. Symptoms start 15–30 minutes after irradiation and last for up to 2 days. Subsequently, there is a 5–10 day period, after which the person dies of infection or internal bleeding. Recovery takes several years and probably is never complete.

Half-life

$$N(t) = N_0 \left(\frac{1}{2} \right)^{\frac{t}{t_{1/2}}}$$

Example

If a radioactive sample initially contains 128 parent nuclei and the half-life, $t_{1/2}$, is 10 years, how many parent nuclei remain after 30 years?

Solution

$$N(30) = 128 \left(\frac{1}{2} \right)^{\frac{30}{10}}$$
$$= 16$$

Black dots represent radioactive nuclei.

Light grey dots are the daughter nuclei.

Nuclear Fission
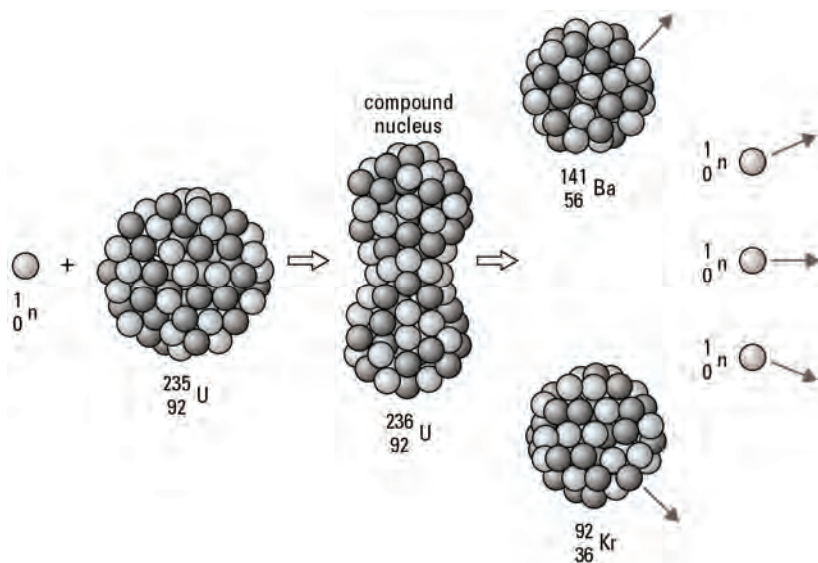
- The uranium nucleus on the left is struck by a moving neutron. The large nucleus is already unstable, and this impact causes the nucleus to vibrate and split into two smaller nuclei.
- Each of the new nuclei is more tightly bound than the original uranium nucleus, so energy is given off.
- Extra neutrons are emitted. These neutrons can go on to produce more fission reactions and, if conditions are right, a *chain reaction*.

What Powers the Sun?
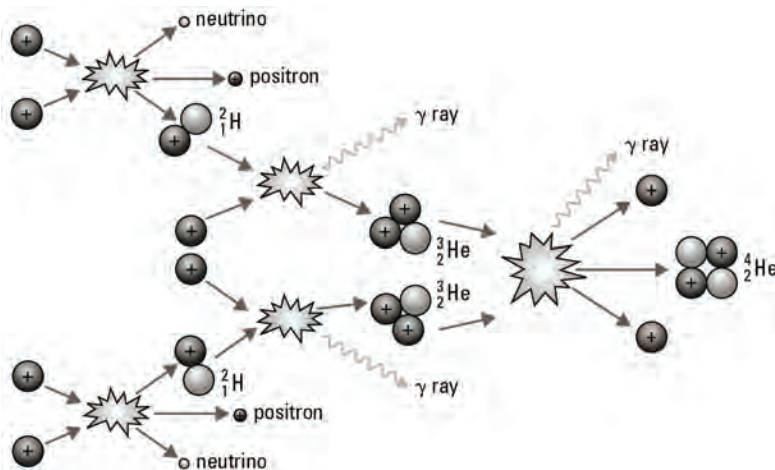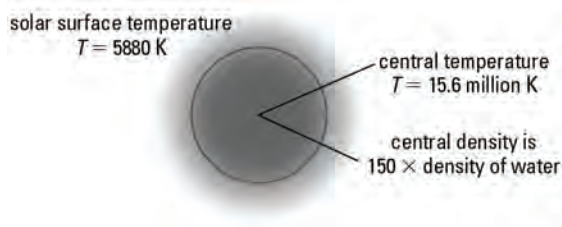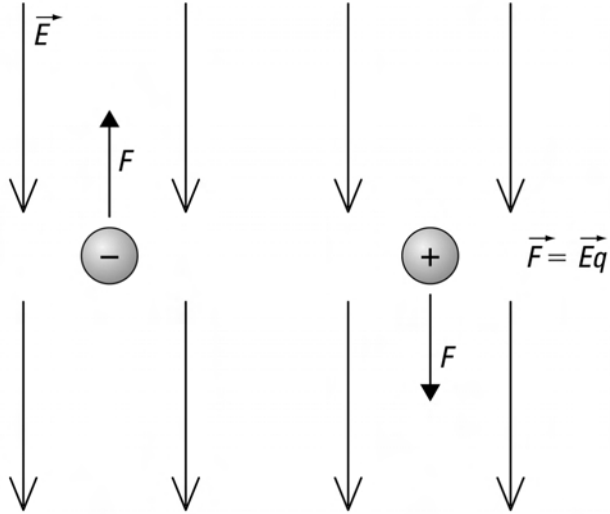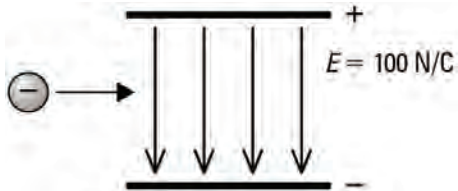
The Proton-proton Cycle
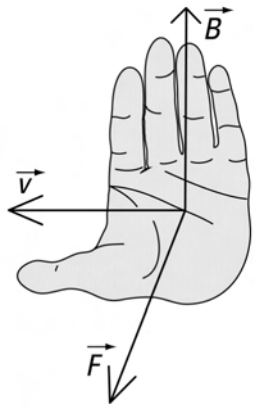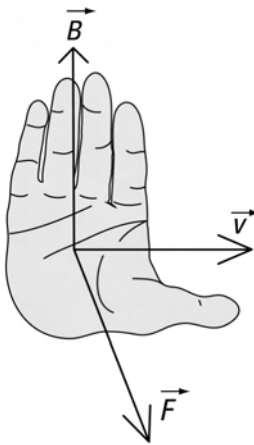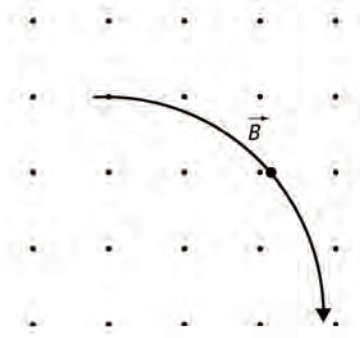
- Four hydrogen nuclei are converted to one helium nucleus.
- Energy is released in the fusion process.
- This reaction can only occur under the extreme temperature and density found at the centre of the Sun and other stars.

Step	Reaction	Energy Released
1	$2\,^1_1H \rightarrow\,^2_1H +\,^0_1\beta + \nu$ (twice)	0.42 MeV (twice)
2	$^1_1H +\,^2_1H \rightarrow\,^3_2He + \gamma$ (twice)	5.49 MeV (twice)
3	$2\,^3_2He \rightarrow\,^4_2He + 2\,^1_1H + \gamma$	12.85 MeV
Total	$4\,^1_1H \rightarrow\,^4_2He + 2\,^0_1\beta + 2\nu + 3\gamma$	24.67 MeV

Review of Motion of Charged Particles in an Electric Field

- The direction of an electric field, $\vec{E}$, is defined as the direction in which a positive charge accelerates when placed in the field.
- The magnitude of the electric force is directly proportional to the size of the electric charge:

$$F = \left|\vec{E}\right| q$$

- The magnitude of the acceleration on a charged particle in an electric field is:

$$F = ma = \left|\vec{E}\right| q$$

$$a = \frac{\left|\vec{E}\right| q}{m}$$

Example

A mass of 0.001 kg and charge 1×10^{-6} C is moving at 1 m/s, as shown in the diagram on the right. Sketch the path of the mass as it travels through the electric field and determine its speed 2.0 s after entering the field.

Review of Motion of Charged Particles in a Magnetic Field

<table>
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The Left-hand Rule for Negatively Charged Particles:

- The thumb points in the direction of motion.
- Fingers point in the direction of the magnetic field.
- The palm points in the direction of the force.

$$F = qBv = m\frac{v^2}{r}$$

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The Right-hand Rule for Positively Charged Particles:

- The thumb points in the direction of motion.
- Fingers point in the direction of the magnetic field.
- The palm points in the direction of the force.

$$r = \frac{mv}{qB}$$

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Examples

1. How do you know that the particle on the left is charged?

2. Is the charge positive or negative?

3. If $B = 1$ T, $v = 100$ m/s, and $|q| = e$, determine the radius of the curved path.

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Conceptual Overview
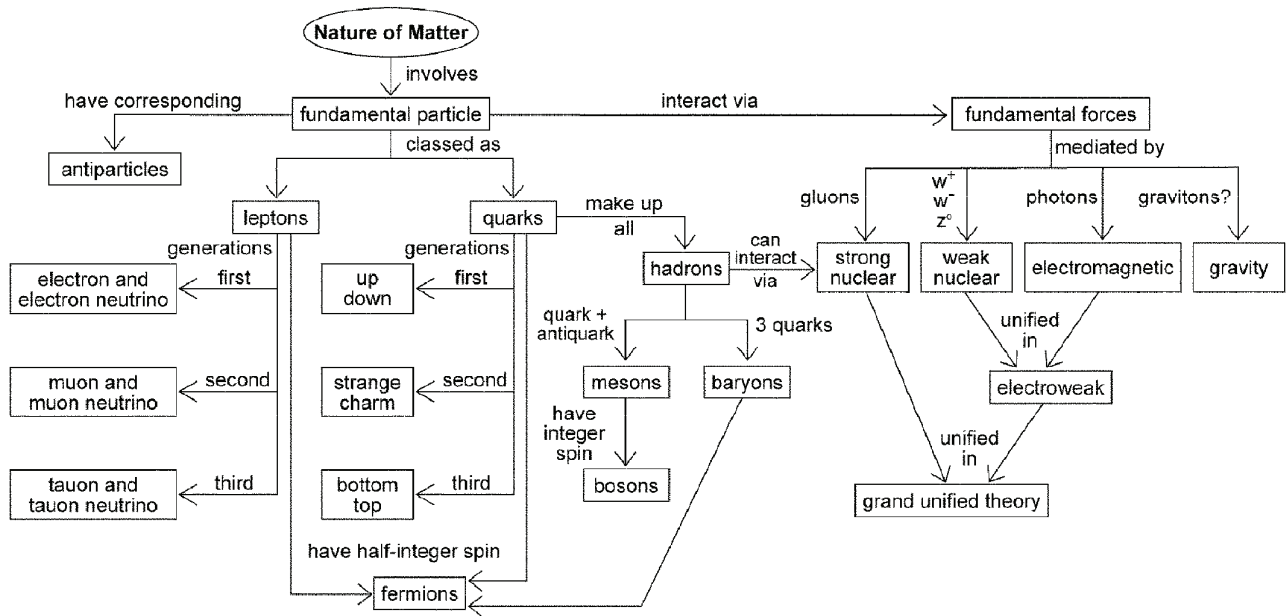
Drawing Momentum Vectors

1. A student on a skateboard has a combined mass of 70 kg. Calculate the momentum of the student-skateboard system if he is travelling west at 1.2 m/s. Draw the momentum vector.

    Given

    $m = 70$ kg $\qquad\qquad\qquad \vec{v} = 1.2$ m/s [W]

    Required
    momentum of system ($\vec{p}$)
    momentum vector diagram

    Analysis and Solution
    The momentum of the student-skateboard system is in the direction of the velocity.
    So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

    $p = mv$
    $\quad = (70$ kg$)(1.2$ m/s$)$
    $\quad = 84$ kg•m/s

    Draw the momentum vector to scale.
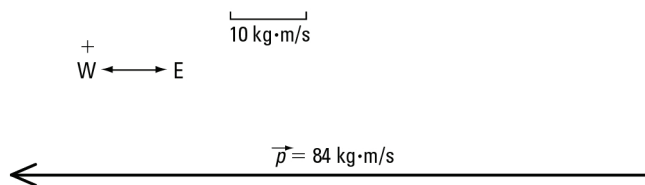
    Paraphrase
    The momentum of the student-skateboard system is 84 kg•m/s [W].

2. An athlete on a sled in the Winter Olympics skeleton event has a velocity of 35 m/s [N]. If the momentum of the athlete-sled system has a magnitude of 2.73×10^3 kg•m/s, what is the combined mass of the athlete and sled? Include a drawing of the momentum vector in your answer.

    Given
    $p = 2.73 \times 10^3$ kg•m/s $\qquad\qquad \vec{v} = 35$ m/s [N]

    Required
    mass of athlete and sled (m)
    momentum vector diagram

    Analysis and Solution
    Draw the momentum vector to scale.

    The momentum of the athlete-sled system is in the direction of
    the velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the mass.

    $p = mv$

    $m = \dfrac{p}{v}$

    $\quad = \dfrac{2.73 \times 10^3 \text{ kg} \cdot \text{m/s}}{35 \text{ m/s}}$

    $\quad = 78$ kg
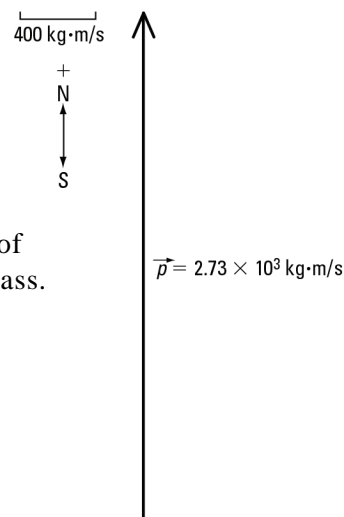
    Paraphrase
    The combined mass of the athlete and sled is 78 kg.

Drawing Momentum Vectors (continued)

3.   A 3.5-kg Canada goose has a momentum of magnitude 49 kg•m/s while flying 36º N of E.
     (a) Use a momentum vector diagram to determine the east and north components.
         Check the accuracy of your measurements by calculation.
     (b) What is the velocity of the goose?

Given
$m = 3.5$ kg $\vec{p} = 49$ kg•m/s [36º N of E]

Required
(a) momentum vector diagram
    east and north components (p_E and p_N)
(b) velocity of goose ($\vec{v}$)

Analysis and Solution
(a) Draw the momentum vector
    to scale.

    From the diagram at right, careful
    measurements give $p_E = 40$ kg•m/s
    and $p_N = 29$ kg•m/s.

5.0 kg·m/s

$\vec{p}$ = 49 kg·m/s

29 kg·m/s

36º
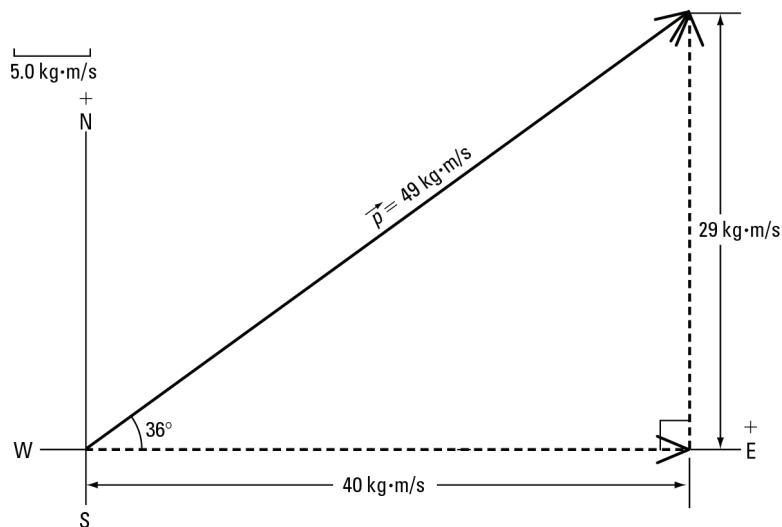
40 kg·m/s

From calculation, $p_E = p \cos 36º$ and $p_N = p \sin 36º$
                    $= (49$ kg•m/s$)(\cos 36º)$ $= (49$ kg•m/s$)(\sin 36º)$
                    $= 40$ kg•m/s $= 29$ kg•m/s
The calculated values agree with the measurements of p_E and p_N obtained from the diagram.

(b) The momentum of the goose is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$
    to find the magnitude of the velocity.

    $p = mv$

    $v = \dfrac{p}{m}$

    $= \dfrac{49 \text{ kg m/s}}{3.5 \text{ kg}}$

    $= 14$ m/s

    $\vec{v} = 14$ m/s [36º N of E]

Paraphrase
(a) The east component of the momentum is 40 kg•m/s and the north component is 29 kg•m/s.
(b) The velocity of the goose is 14 m/s [36º N of E].

Adding Momentum Vectors

1. Railroad car A of mass 10 000 kg is travelling east at 0.15 m/s, when it collides with railroad car B of mass 8000 kg. Car B was initially travelling west at 0.25 m/s. The cars couple together upon impact.
   (a) Use a momentum vector addition diagram to determine the momentum of the coupled cars immediately after impact. Check the accuracy of your measurements by calculation.
   (b) Determine the velocity of the coupled cars immediately after impact.

Given

m_A = 10 000 kg $\qquad\qquad\qquad$ m_B = 8000 kg

$\vec{v}_{A_i}$ = 0.15 m/s [E] $\qquad\qquad$ $\vec{v}_{B_i}$ = 0.25 m/s [W]

Required

(a) final momentum of coupled cars ($\vec{p}_{sys_f}$)

(b) final velocity of coupled cars ($\vec{v}_f$)

Analysis and Solution

(a) Choose both railroad cars as an isolated system.

The cars move together as a unit after collision.

The momentum of each object in the system is in the direction of its velocity.

So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$\begin{aligned}
p_{A_i} &= m_A v_{A_i} \\
&= (10\ 000\text{ kg})(0.15\text{ m/s}) \\
&= 1.5 \times 10^3\text{ kg}\bullet\text{m/s}
\end{aligned}$ $\qquad\qquad$ $\begin{aligned}
p_{B_i} &= m_B v_{B_i} \\
&= (8000\text{ kg})(0.25\text{ m/s}) \\
&= 2.0 \times 10^3\text{ kg}\bullet\text{m/s}
\end{aligned}$

$\vec{p}_{A_i}$ = 1.5 × 10³ kg•m/s [E] $\qquad\qquad$ $\vec{p}_{B_i}$ = 2.0 × 10³ kg•m/s [W]

Draw a momentum vector addition diagram to scale.
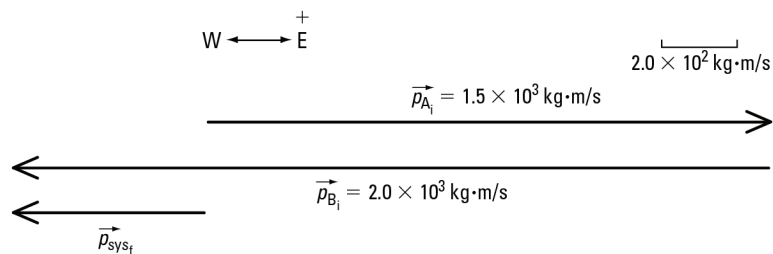
From the diagram above, careful measurements give $\vec{p}_{sys_f}$ = 5.0 × 10² kg•m/s [W].

From calculation, apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$
$$\vec{p}_{A_i} + \vec{p}_{B_i} = \vec{p}_{sys_f}$$
$$m_A \vec{v}_{A_i} + m_B \vec{v}_{B_i} = \vec{p}_{sys_f}$$
$$\begin{aligned}
\vec{p}_{sys_f} &= m_A \vec{v}_{A_i} + m_B \vec{v}_{B_i} \\
&= (10\ 000\text{ kg})(+0.15\text{ m/s}) + (8000\text{ kg})(-0.25\text{ m/s}) \\
&= -5.0 \times 10^2\text{ kg}\bullet\text{m/s}
\end{aligned}$$
$$\vec{p}_{sys_f} = 5.0 \times 10^2\text{ kg}\bullet\text{m/s [W]}$$

The calculated value agrees with the measurement of $\vec{p}_{sys_f}$ obtained from the vector addition diagram.

Adding Momentum Vectors (continued)

(b) The velocity of the coupled cars is in the direction of the momentum of the system.
So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the final velocity.

$$p_{sys_f} = (m_A + m_B)v_f$$

$$v_f = \frac{p_{sys_f}}{m_A + m_B}$$

$$= \frac{5.0 \times 10^2 \text{ kg} \cdot \text{m/s}}{10\ 000 \text{ kg} + 8000 \text{ kg}}$$

$$= \frac{5.0 \times 10^2 \ \cancel{\text{kg}} \text{ m/s}}{18\ 000 \ \cancel{\text{kg}}}$$

$$= 2.8 \times 10^{-2} \text{ m/s}$$

$$\vec{v}_f = 2.8 \times 10^{-2} \text{ m/s [W]}$$

Paraphrase

(a) The momentum of the coupled cars immediately after impact is 5.0×10^2 kg•m/s [W].

(b) The velocity of the coupled cars immediately after impact is 2.8×10^{-2} m/s [W].

2. In an archery competition, a pumpkin that is suspended from a long cord is swinging north at 0.40 m/s when it is struck by a 0.050-kg arrow. The arrow becomes embedded in the pumpkin upon impact. The mass of the pumpkin is 3.0 kg, and the arrow was initially travelling east at 32 m/s.
    (a) Use a momentum vector addition diagram to determine the momentum of the arrow-pumpkin system immediately after impact. Check the accuracy of your measurements by calculation.
    (b) Determine the velocity of the arrow-pumpkin system immediately after impact.

Given

$m_a = 0.050$ kg $m_p = 3.0$ kg

$\vec{v}_{a_i} = 32$ m/s [E] $\vec{v}_{p_i} = 0.40$ m/s [N]

Required

(a) final momentum of system ($\vec{p}_{sys_f}$)

(b) final velocity of system ($\vec{v}_f$)

Analysis and Solution

(a) Choose the arrow and pumpkin as an isolated system.
The arrow and pumpkin move together as a unit after collision.
The momentum of each object in the system is in the direction of its velocity.
So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$p_{a_i} = m_a v_{a_i}$ $p_{p_i} = m_p v_{p_i}$

$\quad = (0.050 \text{ kg})(32 \text{ m/s})$ $\quad = (3.0 \text{ kg})(0.40 \text{ m/s})$

$\quad = 1.6 \text{ kg} \cdot \text{m/s}$ $\quad = 1.2 \text{ kg} \cdot \text{m/s}$

$\vec{p}_{a_i} = 1.6$ kg•m/s [E] $\vec{p}_{p_i} = 1.2$ kg•m/s [N]

Adding Momentum Vectors (continued)

Draw a momentum vector addition diagram to scale.
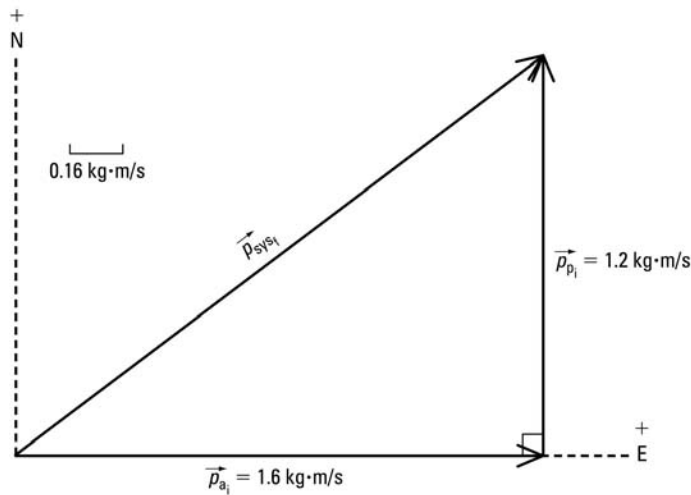
From the diagram above, careful measurements give $\vec{p}_{\text{sys}_f} = 2.0$ kg•m/s [53º E of N].

From calculation, first resolve all velocities into east and north components.

Vector	East component	North component
$\vec{v}_{a_i}$	32 m/s	0
$\vec{v}_{p_i}$	0	0.40 m/s

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$p_{\text{sys}_{iE}} = p_{\text{sys}_{fE}}$$

$$p_{a_{iE}} + p_{p_{iE}} = p_{\text{sys}_{fE}}$$

$$m_a v_{a_{iE}} + 0 = p_{\text{sys}_{fE}}$$

$$p_{\text{sys}_{fE}} = m_a v_{a_{iE}}$$

$$= (0.050 \text{ kg})(32 \text{ m/s})$$

$$= 1.6 \text{ kg•m/s}$$

N direction

$$p_{\text{sys}_{iN}} = p_{\text{sys}_{fN}}$$

$$p_{a_{iN}} + p_{p_{iN}} = p_{\text{sys}_{fN}}$$

$$0 + m_p v_{p_{iN}} = p_{\text{sys}_{fN}}$$

$$p_{\text{sys}_{fN}} = m_p v_{p_{iN}}$$

$$= (3.0 \text{ kg})(0.40 \text{ m/s})$$

$$= 1.2 \text{ kg•m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{p}_{\text{sys}_f}$.

$$p_{\text{sys}_f} = \sqrt{\left(p_{\text{sys}_{fE}}\right)^2 + \left(p_{\text{sys}_{fN}}\right)^2}$$

$$= \sqrt{(1.6 \text{ kg•m/s})^2 + (1.2 \text{ kg•m/s})^2}$$

$$= 2.0 \text{ kg•m/s}$$

Adding Momentum Vectors (continued)

Use the tangent function to find the direction of $\vec{p}_{sys_f}$.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{1.2 \text{ kg·m/s}}{1.6 \text{ kg·m/s}}$$

$$= 0.7500$$

$$\theta = \tan^{-1}(0.7500)$$

$$= 37°$$

From the figure below, θ is between $\vec{p}_{sys_f}$ and the east direction. So the direction of $\vec{p}_{sys_f}$ measured from north is $90° - 37° = 53°$.
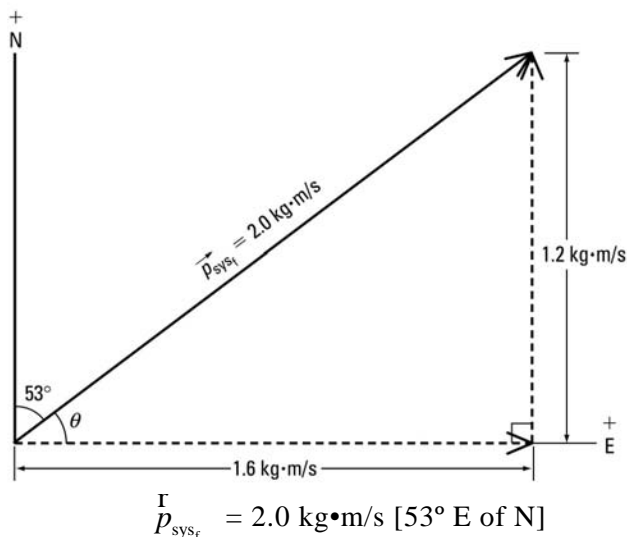
$$\vec{p}_{sys_f} = 2.0 \text{ kg•m/s [53° E of N]}$$

The calculated value agrees with the measurement of $\vec{p}_{sys_f}$ obtained from the vector

addition diagram.

(b) The velocity of the arrow-pumpkin system is in the direction of the momentum of the system. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the final velocity.

$$p_{sys_f} = (m_a + m_p)v_f$$

$$v_f = \frac{p_{sys_f}}{m_a + m_p}$$

$$= \frac{2.00 \text{ kg•m/s}}{0.050 \text{ kg} + 3.0 \text{ kg}}$$

$$= \frac{2.00 \text{ kg m/s}}{3.050 \text{ kg}}$$

$$= 0.66 \text{ m/s}$$

$$\vec{v}_f = 0.66 \text{ m/s [53° E of N]}$$

Paraphrase

(a) The momentum of the arrow-pumpkin system immediately after impact is 2.0 kg•m/s [53° E of N].

(b) The velocity of the system immediately after impact is 0.66 m/s [53° E of N].

Three-body Collisions

1. Two identical curling stones (B and C) each of mass 19.6 kg are stationary and touching each other on the ice. Another curling stone (A) moving at 1.80 m/s [S] strikes stone B off centre, and stone A moves off at 1.00 m/s [46.0° W of S] while stone B travels at 1.10 m/s [78.0° E of S]. What will be the velocity of stone C immediately after the collision? Ignore frictional and rotational effects.

Given

$m_A = m_B = m_C = 19.6$ kg

$\vec{v}_{A_i} = 1.80$ m/s [S] $\vec{v}_{B_i} = 0$ m/s $\vec{v}_{C_i} = 0$ m/s

$\vec{v}_{A_f} = 1.00$ m/s [46.0° W of S] $\vec{v}_{B_f} = 1.10$ m/s [78.0° E of S]
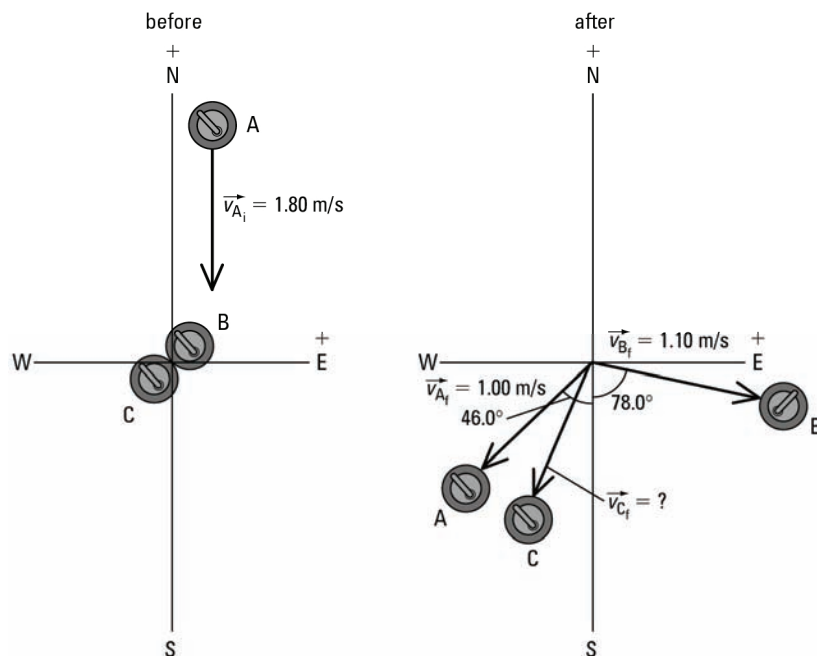
Required

final velocity of stone C ($\vec{v}_{C_f}$)

Analysis and Solution

Choose all three curling stones as an isolated system.

Stones B and C have initial velocities of zero. So their initial momenta are zero.

$$\vec{p}_{B_i} = \vec{p}_{C_i} = 0$$

Resolve all velocities into east and north components.

Vector	East component	North component
$\vec{v}_{A_i}$	0	−1.80 m/s
$\vec{v}_{B_i}$	0	0
$\vec{v}_{C_i}$	0	0
$\vec{v}_{A_f}$	−(1.00 m/s)(sin 46.0°)	−(1.00 m/s)(cos 46.0°)
$\vec{v}_{B_f}$	(1.10 m/s)(sin 78.0°)	−(1.10 m/s)(cos 78.0°)

Three-body Collisions (continued)
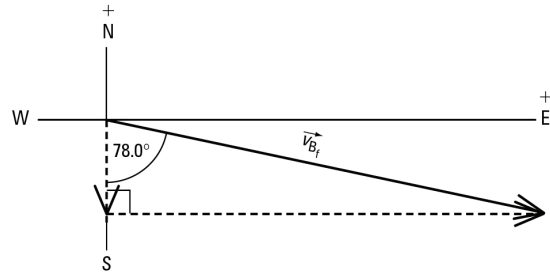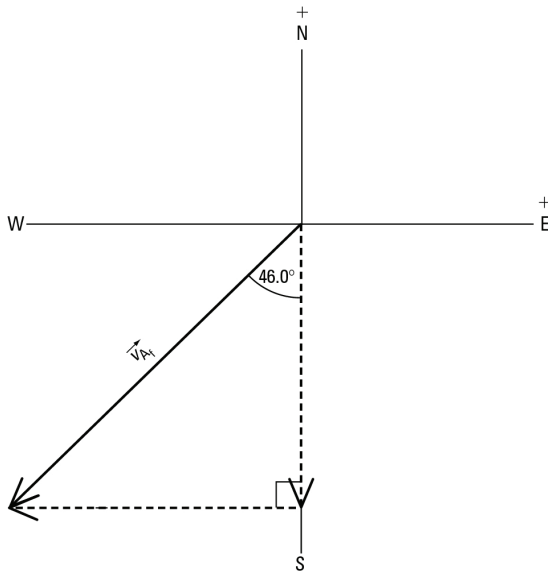
Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$p_{\text{sys}_{iE}} = p_{\text{sys}_{fE}}$$

$$p_{A_{iE}} + p_{B_{iE}} + p_{C_{iE}} = p_{A_{fE}} + p_{B_{fE}} + p_{C_{fE}}$$

$$m_A v_{A_{iE}} + 0 + 0 = m_A v_{A_{fE}} + m_B v_{B_{fE}} + m_C v_{C_{fE}}$$

$$m_C v_{C_{fE}} = m_A (v_{A_{iE}} - v_{A_{fE}}) - m_B v_{B_{fE}}$$

$$
\begin{aligned}
v_{C_{fE}} &= \left(\frac{m_A}{m_C}\right)(v_{A_{iE}} - v_{A_{fE}}) - \left(\frac{m_B}{m_C}\right) v_{B_{fE}} \\
&= \left(\frac{19.6\ \cancel{\text{kg}}}{19.6\ \cancel{\text{kg}}}\right)\{0 - [-(1.00\ \text{m/s})(\sin 46.0°)]\} - \left(\frac{19.6\ \cancel{\text{kg}}}{19.6\ \cancel{\text{kg}}}\right)(1.10\ \text{m/s})(\sin 78.0°) \\
&= -0.3566\ \text{m/s}
\end{aligned}
$$

N direction

$$p_{\text{sys}_{iN}} = p_{\text{sys}_{fN}}$$

$$p_{A_{iN}} + p_{B_{iN}} + p_{C_{iN}} = p_{A_{fN}} + p_{B_{fN}} + p_{C_{fN}}$$

$$m_A v_{A_{iN}} + 0 + 0 = m_A v_{A_{fN}} + m_B v_{B_{fN}} + m_C v_{C_{fN}}$$

$$m_C v_{C_{fN}} = m_A (v_{A_{iN}} - v_{A_{fN}}) - m_B v_{B_{fN}}$$

$$
\begin{aligned}
v_{C_{fN}} &= \left(\frac{m_A}{m_C}\right)(v_{A_{iN}} - v_{A_{fN}}) - \left(\frac{m_B}{m_C}\right) v_{B_{fN}} \\
&= \left(\frac{19.6\ \cancel{\text{kg}}}{19.6\ \cancel{\text{kg}}}\right)\{-1.80\ \text{m/s} - [-(1.00\ \text{m/s})(\cos 46.0°)]\} - \left(\frac{19.6\ \cancel{\text{kg}}}{19.6\ \cancel{\text{kg}}}\right)\{-(1.10\ \text{m/s})(\cos 78.0°)\} \\
&= -0.8766\ \text{m/s}
\end{aligned}
$$

Three-body Collisions (continued)

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{C_f}$.

$$v_{C_f} = \sqrt{\left(v_{C_{fE}}\right)^2 + \left(v_{C_{fN}}\right)^2}$$

$$= \sqrt{(-0.3566 \text{ m/s})^2 + (-0.8766 \text{ m/s})^2}$$

$$= 0.946 \text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{C_f}$.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{0.8766 \text{ m/s}}{0.3566 \text{ m/s}}$$

$$= 2.4582$$

$$\theta = \tan^{-1}(2.4582)$$

$$= 67.9°$$

From the figure below, θ is between $\vec{v}_{C_f}$ and the west direction. So the direction of $\vec{v}_{C_f}$ measured from south is $90.0° - 67.9° = 22.1°$.
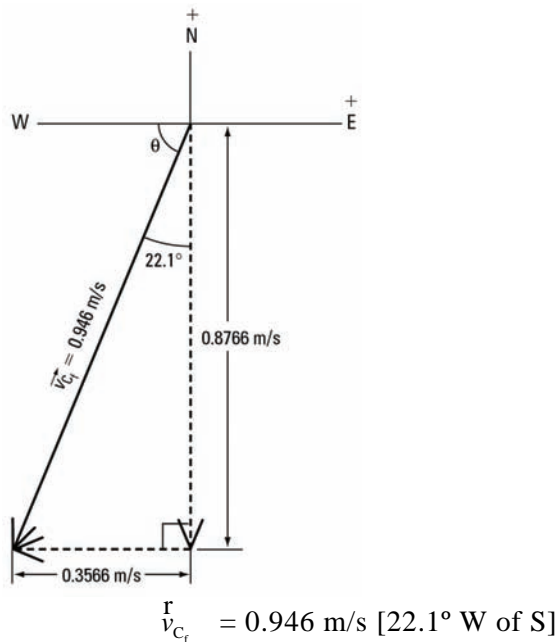
$$\vec{v}_{C_f} = 0.946 \text{ m/s } [22.1° \text{ W of S}]$$

Paraphrase
The final velocity of curling stone C is 0.946 m/s [22.1° W of S].

Three-body Collisions (continued)

2. Inside a helium balloon, helium atom A travelling at 120 m/s [35.0°] and helium atom B travelling at 100 m/s [56.0°] simultaneously collide with stationary helium atom C. Immediately after collision, atom A moves off at 110 m/s [85.8°] and atom B at 95 m/s [18.0°]. What will be the velocity of atom C immediately after the collision? The mass of each helium atom is 6.65×10^{-27} kg.

Given
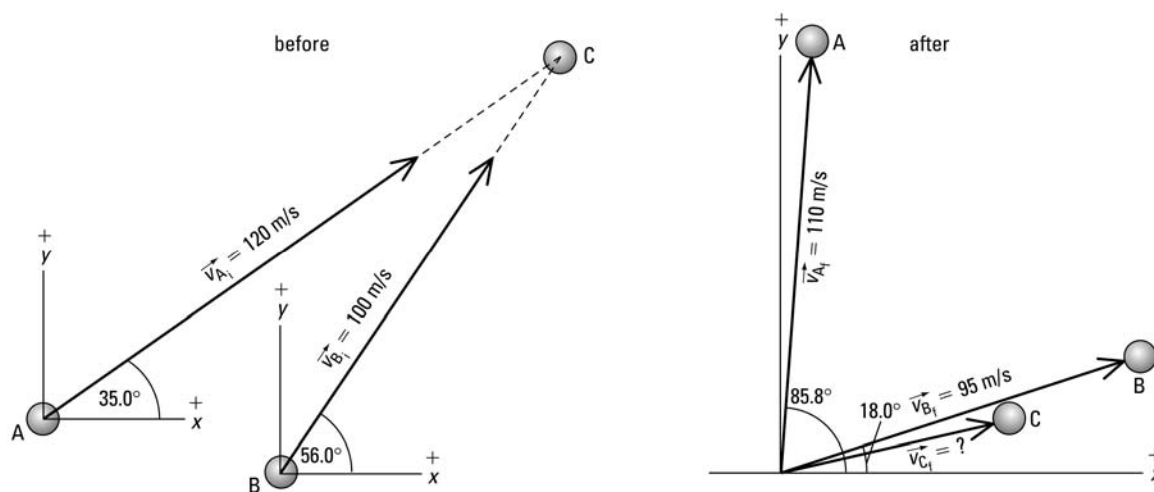
$m_A = m_B = m_C = 6.65 \times 10^{-27}$ kg

$\vec{v}_{A_i} = 120$ m/s [35.0°] $\vec{v}_{B_i} = 100$ m/s [56.0°] $\vec{v}_{C_i} = 0$ m/s

$\vec{v}_{A_f} = 110$ m/s [85.8°] $\vec{v}_{B_f} = 95$ m/s [18.0°]

Required

final velocity of atom C ($\vec{v}_{C_f}$)

Analysis and Solution

Choose all three helium atoms as an isolated system.

Atom C has an initial velocity of zero. So its initial momentum is zero.

$\vec{p}_{C_i} = 0$

Resolve all velocities into *x* and *y* components.

Vector	*x* component	*y* component
$\vec{v}_{A_i}$	(120 m/s)(cos 35.0°)	(120 m/s)(sin 35.0°)
$\vec{v}_{B_i}$	(100 m/s)(cos 56.0°)	(100 m/s)(sin 56.0°)
$\vec{v}_{C_i}$	0	0
$\vec{v}_{A_f}$	(110 m/s)(cos 85.8°)	(110 m/s)(sin 85.8°)
$\vec{v}_{B_f}$	(95 m/s)(cos 18.0°)	(95 m/s)(sin 18.0°)

Three-body Collisions (continued)
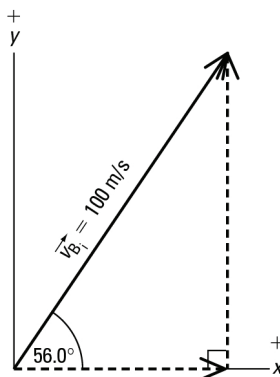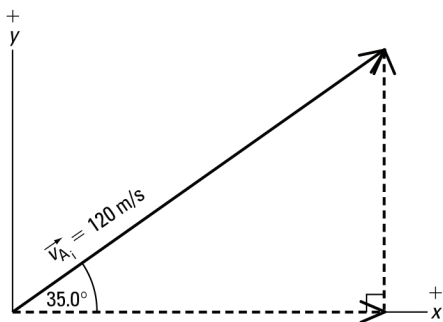
Apply the law of conservation of momentum to the system in the x and y directions.

x direction

$$p_{\text{sys}_{ix}} = p_{\text{sys}_{fx}}$$

$$p_{A_{ix}} + p_{B_{ix}} + p_{C_{ix}} = p_{A_{fx}} + p_{B_{fx}} + p_{C_{fx}}$$

$$m_A v_{A_{ix}} + m_B v_{B_{ix}} + 0 = m_A v_{A_{fx}} + m_B v_{B_{fx}} + m_C v_{C_{fx}}$$

$$m_C v_{C_{fx}} = m_A (v_{A_{ix}} - v_{A_{fx}}) + m_B (v_{B_{ix}} - v_{B_{fx}})$$

$$
\begin{aligned}
v_{C_{fx}} &= \left(\frac{m_A}{m_C}\right)(v_{A_{ix}} - v_{A_{fx}}) + \left(\frac{m_B}{m_C}\right)(v_{B_{ix}} - v_{B_{fx}}) \\[2ex]
&= \left(\frac{6.65 \times 10^{-27}\ \text{kg}}{6.65 \times 10^{-27}\ \text{kg}}\right)\{(120\ \text{m/s})(\cos 35.0°) - (110\ \text{m/s})(\cos 85.8°)\} \\[2ex]
&\quad + \left(\frac{6.65 \times 10^{-27}\ \text{kg}}{6.65 \times 10^{-27}\ \text{kg}}\right)\{(100\ \text{m/s})(\cos 56.0°) - (95\ \text{m/s})(\cos 18.0°)\} \\[2ex]
&= 55.8\ \text{m/s}
\end{aligned}
$$

y direction

$$p_{\text{sys}_{iy}} = p_{\text{sys}_{fy}}$$

$$p_{A_{iy}} + p_{B_{iy}} + p_{C_{iy}} = p_{A_{fy}} + p_{B_{fy}} + p_{C_{fy}}$$

$$m_A v_{A_{iy}} + m_B v_{B_{iy}} + 0 = m_A v_{A_{fy}} + m_B v_{B_{fy}} + m_C v_{C_{fy}}$$

$$m_C v_{C_{fy}} = m_A (v_{A_{iy}} - v_{A_{fy}}) + m_B (v_{B_{iy}} - v_{B_{fy}})$$

$$
\begin{aligned}
v_{C_{fy}} &= \left(\frac{m_A}{m_C}\right)(v_{A_{iy}} - v_{A_{fy}}) + \left(\frac{m_B}{m_C}\right)(v_{B_{iy}} - v_{B_{fy}}) \\[2ex]
&= \left(\frac{6.65 \times 10^{-27}\ \text{kg}}{6.65 \times 10^{-27}\ \text{kg}}\right)\{(120\ \text{m/s})(\sin 35.0°) - (110\ \text{m/s})(\sin 85.8°)\} \\[2ex]
&\quad + \left(\frac{6.65 \times 10^{-27}\ \text{kg}}{6.65 \times 10^{-27}\ \text{kg}}\right)\{(100\ \text{m/s})(\sin 56.0°) - (95\ \text{m/s})(\sin 18.0°)\} \\[2ex]
&= 12.7\ \text{m/s}
\end{aligned}
$$

Three-body Collisions (continued)

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{C_f}$.

$$v_{C_f} = \sqrt{\left(v_{C_{fx}}\right)^2 + \left(v_{C_{fy}}\right)^2}$$

$$= \sqrt{(55.8 \text{ m/s})^2 + (12.7 \text{ m/s})^2}$$

$$= 57 \text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{C_f}$.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{12.7 \text{ m/s}}{55.8 \text{ m/s}}$$

$$= 0.2270$$

$$\theta = \tan^{-1}(0.2270)$$

$$= 12.8°$$

From the figure below, θ is between $\vec{v}_{C_f}$ and the positive x-axis.

$$\vec{v}_{C_f} = 57 \text{ m/s} [12.8°]$$
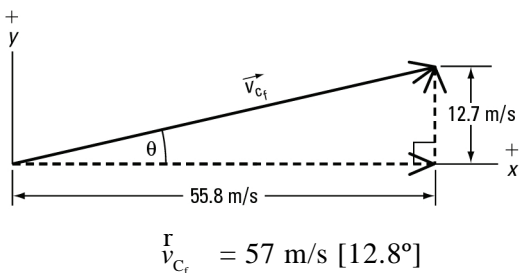
Paraphrase

The final velocity of helium atom C is 57 m/s [12.8°].

Three-body Collisions (continued)

3. In front of a hockey net, a 75.0-kg centre moving at 2.50 m/s [60.0º S of E] collides with an 85.0-kg defenceman moving at 1.80 m/s [20.0º S of W]. They entangle and slide into the stationary 80.0-kg goalie, pushing him back into the crease at 1.50 m/s [74.0º S of W]. What will be the velocity of the two entangled players immediately after colliding with the goalie? Ignore any friction with the ice and rotational effects.

Given

$m_c = 75.0$ kg $m_d = 85.0$ kg $m_g = 80.0$ kg

$\vec{v}_{c_i} = 2.50$ m/s [60.0º S of E] $\vec{v}_{d_i} = 1.80$ m/s [20.0º S of W] $\vec{v}_{g_i} = 0$ m/s

$\vec{v}_{g_f} = 1.50$ m/s [74.0º S of W]
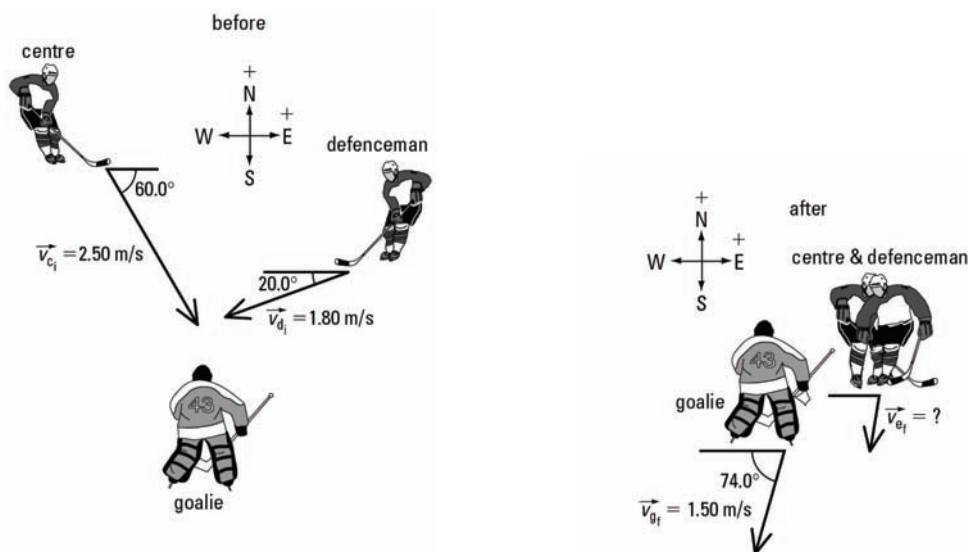
Required

final velocity of entangled players after collision with goalie ($\vec{v}_{e_f}$)

Analysis and Solution

Choose the centre, the defenceman, and the goalie as an isolated system.
There are two collisions in this problem: The first involves the centre and defenceman, the second involves the entangled players and the goalie.
After the first collision, the centre and defenceman move as a unit.

Resolve all velocities related to the first collision into east and north components.

Vector	East component	North component
$\vec{v}_{c_i}$	(2.50 m/s)(cos 60.0º)	−(2.50 m/s)(sin 60.0º)
$\vec{v}_{d_i}$	−(1.80 m/s)(cos 20.0º)	−(1.80 m/s)(sin 20.0º)

Three-body Collisions (continued)
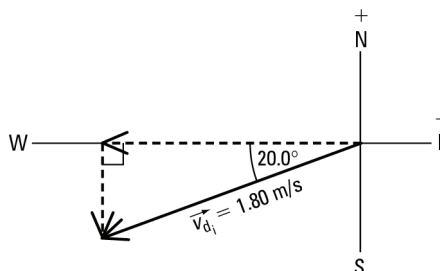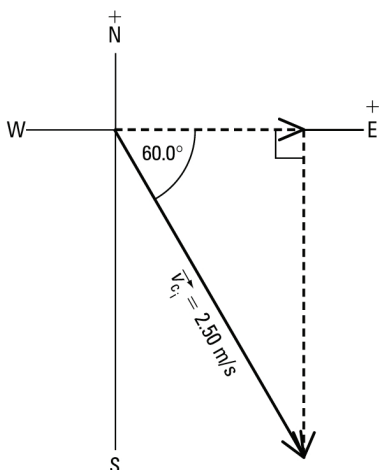
Apply the law of conservation of momentum to the centre and defenceman in the east and north directions.

E direction

$$p_{\text{sys}_{iE}} = p_{\text{sys}_{fE}}$$

$$p_{c_{iE}} + p_{d_{iE}} = p_{\text{sys}_{fE}}$$

$$m_c v_{c_{iE}} + m_d v_{d_{iE}} = (m_c + m_d) v_{f_E}$$

$$v_{f_E} = \left(\frac{1}{m_c + m_d}\right)(m_c v_{c_{iE}} + m_d v_{d_{iE}})$$

$$= \left(\frac{1}{75.0 \text{ kg} + 85.0 \text{ kg}}\right)\{(75.0 \text{ kg})(2.50 \text{ m/s})(\cos 60.0°) + [-(85.0 \text{ kg})(1.80 \text{ m/s})(\cos 20.0°)]\}$$

$$= -0.3126 \text{ m/s}$$

N direction

$$p_{\text{sys}_{iN}} = p_{\text{sys}_{fN}}$$

$$p_{c_{iN}} + p_{d_{iN}} = p_{\text{sys}_{fN}}$$

$$m_c v_{c_{iN}} + m_d v_{d_{iN}} = (m_c + m_d) v_{f_N}$$

$$v_{f_N} = \left(\frac{1}{m_c + m_d}\right)(m_c v_{c_{iN}} + m_d v_{d_{iN}})$$

$$= \left(\frac{1}{75.0 \text{ kg} + 85.0 \text{ kg}}\right)\{-(75.0 \text{ kg})(2.50 \text{ m/s})(\sin 60.0°) + [-(85.0 \text{ kg})(1.80 \text{ m/s})(\sin 20.0°)]\}$$

$$= -1.342 \text{ m/s}$$

Throughout the second collision, the centre and defenceman remain entangled and move as a unit.
The goalie has an initial velocity of zero. So his initial momentum is zero.

$$\vec{p}_{g_i} = 0$$

Three-body Collisions (continued)

Resolve all velocities related to the second collision into east and north components.

| Vector	East component	North component
$\vec{v}_{e_i}$	–0.3126 m/s	–1.342 m/s
$\vec{v}_{g_i}$	0	0
$\vec{v}_{g_f}$	–(1.50 m/s)(cos 74.0º)	–(1.50 m/s)(sin 74.0º)

Apply the law of conservation of momentum to the entangled players and the goalie in the east and north directions.

E direction

$$p_{sys_{iE}} = p_{sys_{fE}}$$

$$p_{e_{iE}} + p_{g_{iE}} = p_{e_{fE}} + p_{g_{fE}}$$

$$m_e\, v_{e_{iE}} + 0 = m_e\, v_{e_{fE}} + m_g\, v_{g_{fE}}$$

$$m_e\, v_{e_{fE}} = m_e\, v_{e_{iE}} - m_g\, v_{g_{fE}}$$

$$v_{e_{fE}} = v_{e_{iE}} - \left(\frac{m_g}{m_e}\right) v_{g_{fE}}$$

$$= -0.3126 \text{ m/s} - \left(\frac{80.0 \text{ kg}}{75.0 \text{ kg} + 85.0 \text{ kg}}\right)\{-(1.50 \text{ m/s})(\cos 74.0°)\}$$

$$= -0.3126 \text{ m/s} - \left(\frac{80.0 \text{ kg}}{160.0 \text{ kg}}\right)\{-(1.50 \text{ m/s})(\cos 74.0°)\}$$

$$= -0.1059 \text{ m/s}$$

N direction

$$p_{sys_{iN}} = p_{sys_{fN}}$$

$$p_{e_{iN}} + p_{g_{iN}} = p_{e_{fN}} + p_{g_{fN}}$$

$$m_e\, v_{e_{iN}} + 0 = m_e\, v_{e_{fN}} + m_g\, v_{g_{fN}}$$

$$m_e\, v_{e_{fN}} = m_e\, v_{e_{iN}} - m_g\, v_{g_{fN}}$$

$$v_{e_{fN}} = v_{e_{iN}} - \left(\frac{m_g}{m_e}\right) v_{g_{fN}}$$

$$= -1.342 \text{ m/s} - \left(\frac{80.0 \text{ kg}}{75.0 \text{ kg} + 85.0 \text{ kg}}\right)\{-(1.50 \text{ m/s})(\sin 74.0°)\}$$

$$= -1.342 \text{ m/s} - \left(\frac{80.0 \text{ kg}}{160.0 \text{ kg}}\right)\{-(1.50 \text{ m/s})(\sin 74.0°)\}$$

$$= -0.6210 \text{ m/s}$$
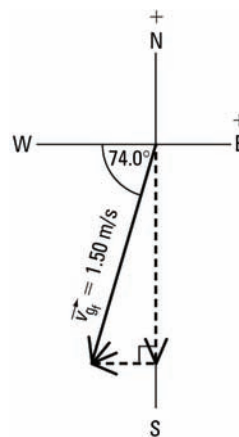
Three-body Collisions (continued)

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{e_f}$.

$$
\begin{aligned}
v_{e_f} &= \sqrt{\left(v_{e_{fE}}\right)^2 + \left(v_{e_{fN}}\right)^2} \\
&= \sqrt{(-0.1059 \text{ m/s})^2 + (-0.6210 \text{ m/s})^2} \\
&= 0.630 \text{ m/s}
\end{aligned}
$$

Use the tangent function to find the direction of $\vec{v}_{e_f}$.

$$
\begin{aligned}
\tan \theta &= \frac{\text{opposite}}{\text{adjacent}} \\
&= \frac{0.6210 \text{ m/s}}{0.1059 \text{ m/s}} \\
&= 5.8630 \\
\theta &= \tan^{-1}(5.8630) \\
&= 80.3°
\end{aligned}
$$

From the figure below, θ is between $\vec{v}_{e_f}$ and the west direction.

$$
\vec{v}_{e_f} = 0.630 \text{ m/s [80.3° S of W]}
$$
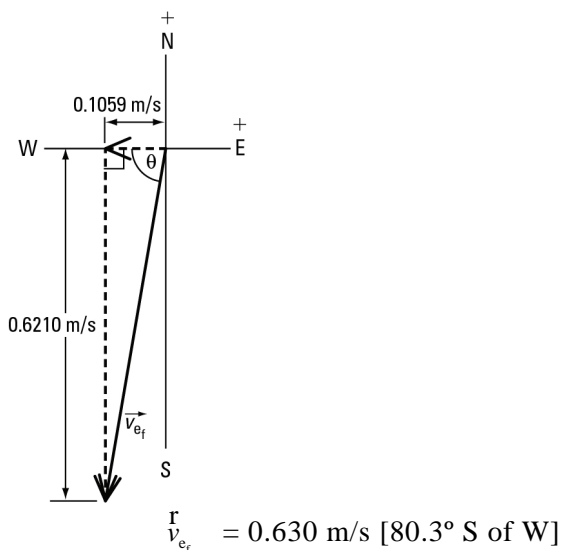
Paraphrase
The final velocity of the entangled players after colliding with the goalie is 0.630 m/s
[80.3° S of W].