Lesson 6: Manipulating Equations

Manipulating equations is probably one of the most important skills to master in a high school physics course.

- Although it is based on familiar (and fairly simple) math concepts, it is still a stumbling block for most new physics students.
- Manipulating an equation means that you rearrange the equation so that the unknown you are trying to calculate is on its own on one side of the equation.
- Later in the course it will also give you the power to combine formulas (which is necessary) to solve more complicated problems.
- Learning how to manipulate formulas now (while the formulas are still easy ones!) will pay off later.

At all times remember two basic rules from math...

- 1. To move something to the other side, just do the opposite math operation to it.
- 2. If you do it to one side, do it to the other.

Warning! Do not use any "the like triangle" that you learned in Junior High for the formula v = d/t. This works great for an formula, but try using it for $v_f^2 = v_i^2 + 2ad...$ it does not work!

Example 1: The basic formula for calculating the velocity of an object is $\mathbf{v} = \mathbf{d} / \mathbf{t}$, where "v" is the velocity, "d" is the displacement, and "t" is the time. This formula is great "as-is" if we are going to calculate velocity, but what if I need to calculate the displacement and I've been given the velocity and time? **Solve** the formula to solve for " \mathbf{d} ".

In the formula v = d / t, "d" is being divided by "t". To get "t" to the other side, we need to do the opposite... multiply by "t"!

$$v = \frac{d}{t}$$

$$v = \frac{d}{t}(t)$$

but what we do to one side, we do to the other...

$$(t) v = \frac{d}{t}(t)$$

the "t" on the right side cancel each other out leaving...

(t)
$$v = d$$

the last step (and it's basically just a tradition) is to put our unknown on the left side of the equation. So let's flip flop the whole thing to get our final equation!

$$d = t v$$

And that's it! This is not a new formula you need to memorize; it is a formula already on your data sheet that you have manipulated to use more easily for a particular question.

Now try to solve the same formula for "t". You should get...

$$t = \frac{d}{v}$$

Be careful with formulas with addition, subtraction, square roots and squares.

- You basically need to follow the **BEDMAS** (Brackets, Exponents, Division, Multiplication, Addition, Subtraction) rule from math, but backwards.
- Usually take care of any addition and subtraction first, then multiplication and division, and finally exponents (remember, square root is just an exponent).

Example 2: Solve the formula " $v_f^2 = v_i^2 + 2ad$ " for v_i

Before doing anything else, take care of anything being added or subtracted to v_i by doing the opposite...

$$v_f^2$$
 - (2ad) = v_i^2 +2ad - (2ad)

which leaves us with...

$$v_f^2 - 2ad = v_i^2$$

flip the whole formula (so v_i is on the left) and take the square root of both sides...

$$v_i = \sqrt{v_f^2 - 2ad}$$

and you're done!